

12-1.

A baseball is thrown downward from a 50-ft tower with an initial speed of 18 ft/s. Determine the speed at which it hits the ground and the time of travel.

SOLUTION

$$(+\downarrow) \quad v_2^2 = v_1^2 + 2a_c(s_2 - s_1)$$

$$v_2^2 = (18)^2 + 2(32.2)(50 - 0)$$

$$v_2 = 59.532 = 59.5 \text{ ft/s } \downarrow$$

Ans.

$$(+\downarrow) \quad v_2 = v_1 + a_c t$$

$$59.532 = 18 + 32.2(t)$$

$$t = 1.29 \text{ s}$$

Ans.**Ans:**

$$v_2 = 59.5 \text{ ft/s}$$

$$t = 1.29 \text{ s}$$

12-2.

Starting from rest, a particle moving in a straight line has an acceleration of $a = (2t - 6) \text{ m/s}^2$, where t is in seconds. What is the particle's velocity when $t = 6 \text{ s}$, and what is its position when $t = 11 \text{ s}$?

SOLUTION

$$a = 2t - 6$$

$$dv = a \, dt$$

$$\int_0^v dv = \int_0^t (2t - 6) \, dt$$

$$v = t^2 - 6t$$

$$ds = v \, dt$$

$$\int_0^s ds = \int_0^t (t^2 - 6t) \, dt$$

$$s = \frac{t^3}{3} - 3t^2$$

When $t = 6 \text{ s}$,

$$v = 0$$

Ans.

When $t = 11 \text{ s}$,

$$s = 80.7 \text{ m}$$

Ans.

Ans:

$$v = 0$$

$$s = 80.7 \text{ m}$$

12-3.

From approximately what floor of a building must a car be dropped from an at-rest position so that it reaches a speed of 80.7 ft/s (55 mi/h) when it hits the ground? Each floor is 12 ft higher than the one below it. (*Note:* You may want to remember this when traveling 55 mi/h.)

SOLUTION

$$(+\downarrow) \quad v^2 = v_0^2 + 2a_c(s - s_0)$$

$$80.7^2 = 0 + 2(32.2)(s - 0)$$

$$s = 101.13 \text{ ft}$$

$$\# \text{ of floors} = \frac{101.13}{12} = 8.43$$

The car must be dropped from the 9th floor.

Ans.

Ans:
 $n = 9$

***12-4.**

A car is traveling at 15 m/s when the traffic light 50 m ahead turns yellow. Determine the required constant deceleration of the car and the time needed to stop the car at the light.

SOLUTION

Kinematics:

$$v_0 = 0, s_0 = 0, s = 50 \text{ m and } v_0 = 15 \text{ m/s.}$$

$$\left(\begin{array}{l} \pm \\ \rightarrow \end{array} \right) \quad v^2 = v_0^2 + 2a_c(s - s_0)$$

$$0 = 15^2 + 2a_c(50 - 0)$$

$$a_c = -2.25 \text{ m/s}^2 = 2.25 \text{ m/s}^2 \leftarrow$$

Ans.

$$\left(\begin{array}{l} \pm \\ \rightarrow \end{array} \right) \quad v = v_0 + a_c t$$

$$0 = 15 + (-2.25)t$$

$$t = 6.67 \text{ s}$$

Ans.

Ans:

$$a_c = -2.25 \text{ m/s}^2$$

$$t = 6.67 \text{ s}$$

12-5.

The velocity of a particle traveling in a straight line is given by $v = (6t - 3t^2)$ m/s, where t is in seconds. If $s = 0$ when $t = 0$, determine the particle's deceleration and position when $t = 3$ s. How far has the particle traveled during the 3-s time interval, and what is its average speed?

SOLUTION

$$\frac{ds}{dt} = v$$

$$ds = (6t - 3t^2) dt$$

$$\int_0^s ds = \int_0^t (6t - 3t^2) dt$$

$$s = 3t^2 - t^3$$

At $t = 3$ s,

$$s = 3(3)^2 - 3^3 = 0$$

When $v = 0$;

$$0 = 6t - 3t^2 \quad t = 0 \text{ and } 2 \text{ s}$$

At $t = 2$ s,

$$s = 3(2)^2 - 2^3 = 4 \text{ m}$$

From the figure

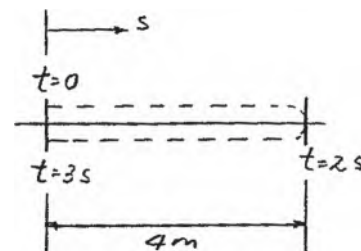
$$s_T = 2(4) = 8 \text{ m}$$

$$(v_{\text{avg}})_{\text{sp}} = \frac{s_T}{\Delta t} = \frac{8}{3} = 2.67 \text{ m/s}$$

$$a = \frac{dv}{dt} = 6 - 6t$$

At $t = 3$ s,

$$a = 6 - 6(3) = -12 \text{ m/s}^2$$



Ans.

Ans.

Ans.

Ans.

Ans:

$$s = 0$$

$$s_T = 8 \text{ m}$$

$$(v_{\text{avg}})_{\text{sp}} = 2.67 \text{ m/s}$$

$$a = -12 \text{ m/s}^2$$

12-6.

A particle moves along a straight line with a velocity $v = (200s) \text{ mm/s}$, where s is in millimeters. Determine the acceleration of the particle at $s = 2000 \text{ mm}$. How long does the particle take to reach this position if $s = 500 \text{ mm}$ when $t = 0$?

SOLUTION**Acceleration:**

$$\left(\frac{\pm}{\rightarrow} \right) \quad \frac{dv}{ds} = 200$$

$$\text{Thus, } a = v \frac{dv}{ds} = (200s)(200) = \{40(10^3)s\} \text{ mm/s}^2$$

When $s = 2000 \text{ mm}$,

$$a = 40(10^3)(2000) = 80(10^6) \text{ mm/s}^2 = 80 \text{ km/s}^2 \rightarrow \quad \textbf{Ans.}$$

Position:

$$\left(\frac{\pm}{\rightarrow} \right) \quad dt = \frac{ds}{v}$$

$$\int_0^t dt = \int_{500 \text{ mm}}^s \frac{ds}{200s}$$

$$t \Big|_0^t = \frac{1}{200} \ln s \Big|_{500 \text{ mm}}^s$$

$$t = \frac{1}{200} \ln \frac{s}{500}$$

At $s = 2000 \text{ mm}$,

$$t = \frac{1}{200} \ln \frac{2000}{500} = 6.93(10^{-3}) \text{ s} = 6.93 \text{ ms} \quad \textbf{Ans.}$$

$$\begin{aligned} \textbf{Ans:} \\ a &= 80 \text{ km/s}^2 \\ t &= 6.93 \text{ ms} \end{aligned}$$

12-7.

A car has an initial speed of 25 m/s and a constant deceleration of 3 m/s^2 . Determine the velocity of the car when $t = 4 \text{ s}$. What is the displacement of the car during the 4-s time interval? How much time is needed to stop the car?

SOLUTION

$$v = v_0 + a_c t$$

$$v = 25 + (-3)(4) = 13 \text{ m/s}$$

Ans.

$$\Delta s = s - s_0 = v_0 t + \frac{1}{2} a_c t^2$$

$$\Delta s = s - 0 = 25(4) + \frac{1}{2}(-3)(4)^2 = 76 \text{ m}$$

Ans.

$$v = v_0 + a_c t$$

$$0 = 25 + (-3)(t)$$

$$t = 8.33 \text{ s}$$

Ans.**Ans:**

$$v = 13 \text{ m/s}$$

$$\Delta s = 76 \text{ m}$$

$$t = 8.33 \text{ s}$$

***12–8.**

A particle is moving along a straight line such that its position is defined by $s = (10t^2 + 20)$ mm, where t is in seconds. Determine (a) the displacement of the particle during the time interval from $t = 1$ s to $t = 5$ s, (b) the average velocity of the particle during this time interval, and (c) the acceleration when $t = 1$ s.

SOLUTION

$$s = 10t^2 + 20$$

$$(a) \quad s|_{1 \text{ s}} = 10(1)^2 + 20 = 30 \text{ mm}$$

$$s|_{5 \text{ s}} = 10(5)^2 + 20 = 270 \text{ mm}$$

$$\Delta s = 270 - 30 = 240 \text{ mm}$$

Ans.

$$(b) \quad \Delta t = 5 - 1 = 4 \text{ s}$$

$$v_{\text{avg}} = \frac{\Delta s}{\Delta t} = \frac{240}{4} = 60 \text{ mm/s}$$

Ans.

$$(c) \quad a = \frac{d^2s}{dt^2} = 20 \text{ mm/s}^2 \quad (\text{for all } t)$$

Ans.

Ans:

$$(a) \quad \Delta s = 240 \text{ mm}$$

$$(b) \quad v_{\text{avg}} = 60 \text{ mm/s}$$

$$(c) \quad a = 20 \text{ mm/s}^2$$

12–9.

A particle moves along a straight path with an acceleration of $a = (5/s) \text{ m/s}^2$, where s is in meters. Determine the particle's velocity when $s = 2 \text{ m}$, if it is released from rest when $s = 1 \text{ m}$.

SOLUTION

$$a \, ds = v \, dv$$

$$\frac{5}{s} \, ds = v \, dv$$

$$5 \int_1^2 \frac{ds}{s} = \int_0^v v \, dv$$

$$5 (\ln s) \Big|_1^2 = \frac{v^2}{2}$$

$$v = 2.63 \text{ m/s}$$

Ans.

Ans:
 $v = 2.63 \text{ m/s}$

12–10.

A particle moves along a straight line with an acceleration of $a = 5/(3s^{1/3} + s^{5/2})$ ft/s², where s is in feet. Determine the particle's velocity when $s = 2$ ft, if it starts from rest when $s = 1$ ft. Use a numerical method to evaluate the integral.

SOLUTION

$$a = \frac{5}{(3s^{1/3} + s^{5/2})}$$

$$a \, ds = v \, dv$$

$$\int_1^2 \frac{5 \, ds}{(3s^{1/3} + s^{5/2})} = \int_0^v v \, dv$$

$$0.8351 = \frac{1}{2} v^2$$

$$v = 1.29 \text{ ft/s}$$

Ans.

Ans:
 $v = 1.29 \text{ ft/s}$

12–11.

A particle is moving along a straight line such that its velocity is defined as $v = (-4s^2) \text{ m/s}$, where s is in meters. If $s = 2 \text{ m}$ when $t = 0$, determine the velocity and acceleration as functions of time.

SOLUTION

$$v = -4s^2$$

$$\frac{ds}{dt} = -4s^2$$

$$\int_2^s s^{-2} ds = \int_0^t -4 dt$$

$$-s^{-1} \Big|_2^s = -4t \Big|_0^t$$

$$t = \frac{1}{4}(s^{-1} - 0.5)$$

$$s = \frac{2}{8t + 1}$$

$$v = -4 \left(\frac{2}{8t + 1} \right)^2 = -\frac{16}{(8t + 1)^2} \text{ m/s}$$

Ans.

$$a = \frac{dv}{dt} = \frac{16(2)(8t + 1)(8)}{(8t + 1)^4} = \frac{256}{(8t + 1)^3} \text{ m/s}^2$$

Ans.**Ans:**

$$v = -\frac{16}{(8t + 1)^2} \text{ m/s}$$

$$a = \frac{256}{(8t + 1)^3} \text{ m/s}^2$$

***12–12.**

The speed of a particle traveling along a straight line within a liquid is measured as a function of its position as $v = (100 - s)$ mm/s, where s is in millimeters. Determine (a) the particle's deceleration when it is located at point A , where $s_A = 75$ mm, (b) the distance the particle travels before it stops, and (c) the time needed to stop the particle.

SOLUTION

(a) $a \, ds = v \, dv$

$$a = v \frac{dv}{ds} = (100 - s)(-1) = s - 100$$

When $s = s_A = 75$ mm, $a = 75 - 100 = -25$ mm/s²

Ans.

(b) $v = 100 - s$

$$0 = 100 - s \quad s = 100 \text{ mm}$$

Ans.

(c) $v = \frac{ds}{dt}$

$$dt = \frac{ds}{v} = \frac{ds}{100 - s}$$

$$\int_0^t dt = \int_0^{100} \frac{ds}{100 - s}$$

$$t = -\ln(100 - s)|_0^{100} \rightarrow \infty$$

Ans.

Ans:

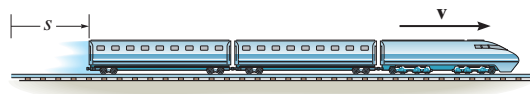
(a) $a_A = -25$ mm/s²

(b) $s = 100$ mm

(c) $t \rightarrow \infty$

12–13.

When a train is traveling along a straight track at 2 m/s, it begins to accelerate at $a = (60v^{-4}) \text{ m/s}^2$, where v is in m/s. Determine its velocity v and its position 3 s after the acceleration.

**SOLUTION**

$$a = \frac{dv}{dt}$$

$$dt = \frac{dv}{a}$$

$$\int_0^3 dt = \int_2^v \frac{dv}{60v^{-4}}$$

$$3 = \frac{1}{300} (v^5 - 32)$$

$$v = 3.925 \text{ m/s} = 3.93 \text{ m/s}$$

Ans.

$$ads = vdv$$

$$ds = \frac{v dv}{a} = \frac{1}{60} v^5 dv$$

$$\int_0^s ds = \frac{1}{60} \int_2^{3.925} v^5 dv$$

$$s = \frac{1}{60} \left(\frac{v^6}{6} \right) \bigg|_2^{3.925}$$

$$= 9.98 \text{ m}$$

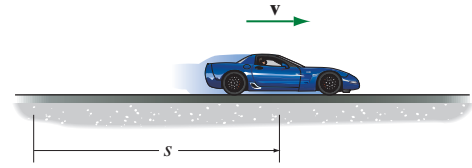
Ans.**Ans:**

$$v = 3.93 \text{ m/s} \rightarrow$$

$$s = 9.98 \text{ m}$$

12-14.

The sports car travels along the straight road such that $v = 3\sqrt{100-s}$ m/s, where s is in meters. Determine the time for the car to reach $s = 60$ m. How much time does it take to stop?

**SOLUTION**

s - t Function: This function can be determined by integrating $ds = v dt$ using the initial condition $s = 0$ at $t = 0$.

$$ds = 3\sqrt{100-s} dt$$

$$\int_0^t dt = \frac{1}{3} \int_0^s \frac{ds}{\sqrt{100-s}}$$

$$t = -\frac{2}{3}(\sqrt{100-s}) \Big|_0^s$$

$$t = \frac{2}{3}(10 - \sqrt{100-s}) \text{ s}$$

When $s = 60$ m,

$$t = \frac{2}{3}(10 - \sqrt{100-60}) = 2.4503 = 2.45 \text{ s} \quad \textbf{Ans.}$$

When the car stops, $v = 0$. Then

$$0 = 3\sqrt{100-s} \quad s = 100 \text{ m}$$

Thus, the required time is

$$t = \frac{2}{3}(10 - \sqrt{100-100}) = 6.6667 = 6.67 \text{ s} \quad \textbf{Ans.}$$

Ans:

$$t|_{s=60 \text{ m}} = 2.45 \text{ s}$$

$$t_{\text{stop}} = 6.67 \text{ s}$$

12–15.

A ball is released from the bottom of an elevator, which is traveling upward with a velocity of 6 ft/s. If the ball strikes the bottom of the elevator shaft in 3 s, determine the height of the elevator from the bottom of the shaft at the instant the ball is released. Also, find the velocity of the ball when it strikes the bottom of the shaft.

SOLUTION

Kinematics: When the ball is released, its velocity will be the same as the elevator at the instant of release. Thus, $v_0 = 6$ ft/s. Also, $t = 3$ s, $s_0 = 0$, $s = -h$, and $a_c = -32.2$ ft/s².

$$(+\uparrow) \quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$-h = 0 + 6(3) + \frac{1}{2} (-32.2)(3^2)$$

$$h = 127 \text{ ft}$$

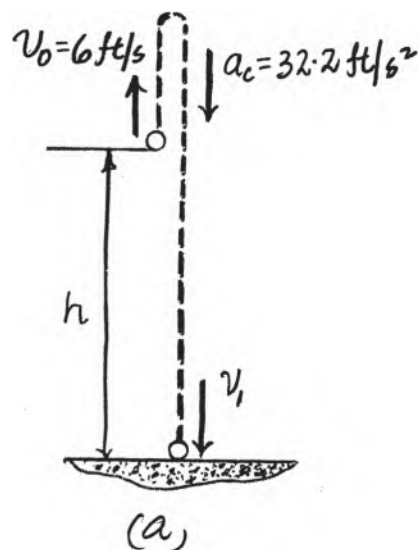
Ans.

$$(+\uparrow) \quad v = v_0 + a_c t$$

$$v = 6 + (-32.2)(3)$$

$$= -90.6 \text{ ft/s} = 90.6 \text{ ft/s} \downarrow$$

Ans.



Ans:

$$h = 127 \text{ ft}$$

$$v = 90.6 \text{ ft/s} \downarrow$$

***12–16.**

A particle is moving along a straight line with an initial velocity of 6 ft/s when it is subjected to a deceleration of $a = (-1.5v^{1/2})$ ft/s², where v is in ft/s. Determine how far it travels before it stops. How much time does this take?

SOLUTION

Distance Traveled:

$$\begin{aligned} ds &= \frac{v dv}{a} \\ \int_0^s ds &= \int_{6 \text{ ft/s}}^v \frac{v}{-1.5v^{\frac{1}{2}}} dv \\ s &= \int_{6 \text{ ft/s}}^v -0.6667 v^{\frac{1}{2}} dv \\ &= \left(-0.4444v^{\frac{3}{2}} + 6.532 \right) \text{ ft} \end{aligned}$$

When $v = 0$, $s = -0.4444 \left(0^{\frac{3}{2}} \right) + 6.532 = 6.53 \text{ ft}$

Ans.

Time:

$$\begin{aligned} dt &= \frac{dv}{a} \\ \int_0^t dt &= - \int_{6 \text{ ft/s}}^v \frac{dv}{1.5v^{\frac{1}{2}}} \\ t &= -1.333 \left(v^{\frac{1}{2}} \right) \Big|_{6 \text{ ft/s}}^v = \left(3.266 - 1.333v^{\frac{1}{2}} \right) \text{ s} \end{aligned}$$

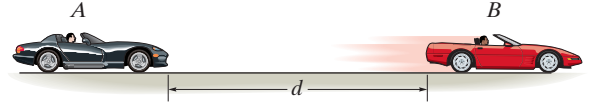
When $v = 0$, $t = 3.266 - 1.333 \left(0^{\frac{1}{2}} \right) = 3.27 \text{ s}$

Ans.

Ans:
 $s = 6.53 \text{ ft}$
 $t = 3.27 \text{ s}$

12–17.

When two cars A and B are next to one another, they are traveling in the same direction with a speed $v_A = v_B$. If B maintains its constant speed, while A begins to decelerate at a_A , determine the distance d between the cars at the instant A stops.

**SOLUTION****Motion of car A:**

$$v = v_0 + a_c t$$

$$0 = v_A - a_A t \quad t = \frac{v_A}{a_A}$$

$$v^2 = v_0^2 + 2a_c(s - s_0)$$

$$0 = v_A^2 + 2(-a_A)(s_A - 0)$$

$$s_A = \frac{v_A^2}{2a_A}$$

Motion of car B:

$$s_B = v_B t = v_B \left(\frac{v_A}{a_A} \right) = \frac{v_A v_B}{a_A}$$

The distance between cars A and B is

$$s_{BA} = |s_B - s_A| = \left| \frac{v_A v_B}{a_A} - \frac{v_A^2}{2a_A} \right| = \frac{2v_A v_B - v_A^2}{2a_A}$$

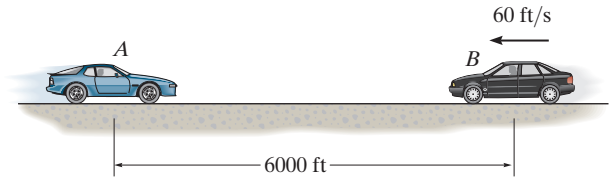
Ans.

Ans:

$$d = \frac{2v_A v_B - v_A^2}{2a_A}$$

12–18.

Car *A* starts from rest at $t = 0$ and travels along a straight road with a constant acceleration of 6 ft/s^2 until it reaches a speed of 80 ft/s . Afterwards it maintains this speed. Also, when $t = 0$, car *B* located 6000 ft down the road is traveling towards *A* at a constant speed of 60 ft/s . Determine the distance traveled by car *A* when they pass each other.



SOLUTION

Distance Traveled: Time for car *A* to achieve $v = 80 \text{ ft/s}$ can be obtained by applying

$$\begin{aligned} \left(\begin{array}{c} \rightarrow \\ \leftarrow \end{array} \right) \quad v &= v_0 + a_c t \\ 80 &= 0 + 6t \\ t &= 13.33 \text{ s} \end{aligned}$$

The distance car *A* travels for this part of motion can be determined by applying

$$\begin{aligned} \left(\begin{array}{c} \rightarrow \\ \leftarrow \end{array} \right) \quad v^2 &= v_0^2 + 2a_c (s - s_0) \\ 80^2 &= 0 + 2(6)(s_1 - 0) \\ s_1 &= 533.33 \text{ ft} \end{aligned}$$

For the second part of motion, car *A* travels with a constant velocity of $v = 80 \text{ ft/s}$ and the distance traveled in $t' = (t_1 - 13.33) \text{ s}$ (t_1 is the total time) is

$$\left(\begin{array}{c} \rightarrow \\ \leftarrow \end{array} \right) \quad s_2 = vt' = 80(t_1 - 13.33)$$

Car *B* travels in the opposite direction with a constant velocity of $v = 60 \text{ ft/s}$ and the distance traveled in t_1 is

$$\left(\begin{array}{c} \rightarrow \\ \leftarrow \end{array} \right) \quad s_3 = vt_1 = 60t_1$$

It is required that

$$\begin{aligned} s_1 + s_2 + s_3 &= 6000 \\ 533.33 + 80(t_1 - 13.33) + 60t_1 &= 6000 \\ t_1 &= 46.67 \text{ s} \end{aligned}$$

The distance traveled by car *A* is

$$s_A = s_1 + s_2 = 533.33 + 80(46.67 - 13.33) = 3200 \text{ ft} \quad \textbf{Ans.}$$

Ans:
 $s_A = 3.2(10^3) \text{ ft}$

12–19.

A car starts from rest and moves with a constant acceleration of 1.5 m/s^2 until it achieves a velocity of 25 m/s . It then travels with constant velocity for 60 seconds. Determine the average speed and the total distance traveled.

SOLUTION

Kinematics: For stage (1) of the motion, $v_0 = 0$, $s_0 = 0$, $v = 25 \text{ m/s}$, and $a_c = 1.5 \text{ m/s}^2$.

$$\begin{aligned} \left(\begin{array}{l} \rightarrow \\ \leftarrow \end{array} \right) \quad v &= v_0 + a_c t \\ 25 &= 0 + 1.5t_1 \\ t_1 &= 16.67 \text{ s} \end{aligned}$$

$$\begin{aligned} \left(\begin{array}{l} \rightarrow \\ \leftarrow \end{array} \right) \quad v^2 &= v_0^2 + 2a_c(s - s_0) \\ 25^2 &= 0 + 2(1.5)(s_1 - 0) \\ s_1 &= 208.33 \text{ m} \end{aligned}$$

For stage (2) of the motion, $s_0 = 208.33 \text{ m}$, $v_0 = 25 \text{ ft/s}$, $t = 60 \text{ s}$, and $a_c = 0$. Thus,

$$\begin{aligned} \left(\begin{array}{l} \rightarrow \\ \leftarrow \end{array} \right) \quad s &= s_0 + v_0 t + \frac{1}{2} a_c t^2 \\ s &= 208.33 + 25(60) + 0 \\ &= 1708.33 \text{ m} = 1708 \text{ m} \end{aligned} \quad \textbf{Ans.}$$

The average speed of the car is then

$$(v_{\text{avg}})_{\text{sp}} = \frac{s}{t_1 + t} = \frac{1708.33}{16.67 + 60} = 22.3 \text{ m/s} \quad \textbf{Ans.}$$

$$\begin{aligned} \textbf{Ans:} \\ s_T &= 1.71 \text{ km} \\ (v_{\text{avg}})_{\text{sp}} &= 22.3 \text{ m/s} \end{aligned}$$

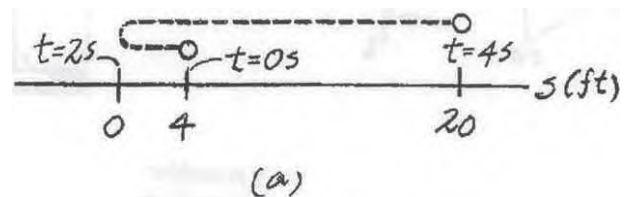
***12–20.**

The velocity of a particle traveling along a straight line is $v = (3t^3 - 6t)$ ft/s, where t is in seconds. If $s = 4$ ft when $t = 0$, determine the position of the particle when $t = 4$ s. What is the total distance traveled during the time interval $t = 0$ to $t = 4$ s? Also, what is the acceleration when $t = 2$ s?

SOLUTION

Position: The position of the particle can be determined by integrating the kinematic equation $ds = v dt$ using the initial condition $s = 4$ ft when $t = 0$. Thus,

$$\begin{aligned} \left(\begin{array}{c} + \\ \rightarrow \end{array} \right) \quad ds &= v dt \\ \int_{4 \text{ ft}}^s ds &= \int_0^t (3t^3 - 6t) dt \\ s \Big|_{4 \text{ ft}}^s &= (t^3 - 3t^2) \Big|_0^t \\ s &= (t^3 - 3t^2 + 4) \text{ ft} \end{aligned}$$



When $t = 4$ s,

$$s|_{t=4 \text{ s}} = 4^3 - 3(4^2) + 4 = 20 \text{ ft} \quad \text{Ans.}$$

The velocity of the particle changes direction at the instant when it is momentarily brought to rest. Thus,

$$\begin{aligned} v &= 3t^2 - 6t = 0 \\ t(3t - 6) &= 0 \\ t &= 0 \text{ and } t = 2 \text{ s} \end{aligned}$$

The position of the particle at $t = 0$ and 2 s is

$$\begin{aligned} s|_0 &= 0^3 - 3(0^2) + 4 = 4 \text{ ft} \\ s|_{2 \text{ s}} &= 2^3 - 3(2^2) + 4 = 0 \end{aligned}$$

Using the above result, the path of the particle shown in Fig. (a) is plotted. From this figure,

$$s_{\text{Tot}} = 4 + 20 = 24 \text{ ft} \quad \text{Ans.}$$

Acceleration:

$$\begin{aligned} \left(\begin{array}{c} + \\ \rightarrow \end{array} \right) \quad a &= \frac{dv}{dt} = \frac{d}{dt} (3t^2 - 6t) \\ a &= (6t - 6) \text{ ft/s}^2 \end{aligned}$$

When $t = 2$ s,

$$a|_{t=2 \text{ s}} = 6(2) - 6 = 6 \text{ ft/s}^2 \rightarrow \quad \text{Ans.}$$

Ans:

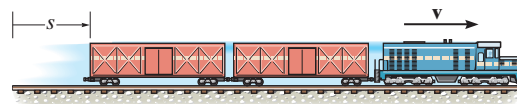
$$s|_{t=4 \text{ s}} = 20 \text{ ft}$$

$$s_T = 24 \text{ ft}$$

$$a|_{t=2 \text{ s}} = 6 \text{ ft/s}^2$$

12–21.

A freight train travels at $v = 60(1 - e^{-t})$ ft/s, where t is the elapsed time in seconds. Determine the distance traveled in three seconds, and the acceleration at this time.

**SOLUTION**

$$v = 60(1 - e^{-t})$$

$$\int_0^s ds = \int_0^3 v dt = \int_0^3 60(1 - e^{-t}) dt$$

$$s = 60(t + e^{-t}) \Big|_0^3$$

$$s = 123 \text{ ft}$$

$$a = \frac{dv}{dt} = 60(e^{-t})$$

$$\text{At } t = 3 \text{ s}$$

$$a = 60e^{-3} = 2.99 \text{ ft/s}^2$$

Ans.**Ans.****Ans:**

$$s_T = 123 \text{ ft}$$

$$a = 2.99 \text{ ft/s}^2 \rightarrow$$

12–22.

The acceleration of the boat is defined by $a = (1.5v^{1/2}) \text{ m/s}^2$, where v is in m/s . Determine its speed when $t = 4 \text{ s}$ if it has a speed of 3 m/s when $t = 0$.

**SOLUTION**

v – t Function: This function can be determined by integrating $dv = a dt$ using the initial condition $v = 3 \text{ m/s}$ at $t = 0$,

$$dv = \frac{3}{2} v^{\frac{1}{2}} dt$$

$$\int_0^t dt = \frac{2}{3} \int_{3 \text{ m/s}}^v \frac{dv}{v^{\frac{1}{2}}}$$

$$t = \frac{4}{3} v^{\frac{1}{2}} \bigg|_{3 \text{ m/s}}^v$$

$$t = \frac{4}{3} \left(v^{\frac{1}{2}} - \sqrt{3} \right)$$

$$v = \left(\frac{3}{4} t + \sqrt{3} \right)^2 \text{ m/s}$$

When $t = 4 \text{ s}$,

$$v = \left[\frac{3}{4}(4) + \sqrt{3} \right]^2 = 22.39 \text{ m/s} = 22.4 \text{ m/s}$$

Ans.

Ans:
 $v = 22.4 \text{ m/s}$

12–23.

A particle is moving along a straight line such that its acceleration is defined as $a = (-2v) \text{ ft/s}^2$, where v is in ft/s . If $v = 20 \text{ ft/s}$ when $s = 0$ and $t = 0$, determine the particle's position, velocity, and acceleration as functions of time.

SOLUTION

$$a = -2v$$

$$\frac{dv}{dt} = -2v$$

$$\int_{20}^v \frac{dv}{v} = \int_0^t -2 dt$$

$$\ln \frac{v}{20} = -2t$$

$$v = (20e^{-2t}) \text{ ft/s}$$

Ans.

$$a = \frac{dv}{dt} = (-40e^{-2t}) \text{ ft/s}^2$$

Ans.

$$\int_0^s ds = v dt = \int_0^t (20e^{-2t}) dt$$

$$s = -10e^{-2t} \Big|_0^t = -10(e^{-2t} - 1)$$

$$s = 10(1 - e^{-2t}) \text{ ft}$$

Ans.**Ans:**

$$v = (20e^{-2t}) \text{ ft/s}$$

$$a = (-40e^{-2t}) \text{ ft/s}^2$$

$$s = 10(1 - e^{-2t}) \text{ ft}$$

***12–24.**

The acceleration of a particle traveling along a straight line is $a = (0.02e^t) \text{ m/s}^2$, where t is in seconds. If $v = 0$, $s = 0$ when $t = 0$, determine the velocity and acceleration of the particle at $s = 4 \text{ m}$.

SOLUTION

Velocity:

$$\left(\pm \right) \quad dv = a \, dt$$

$$\int_0^v dv = \int_0^t 0.02e^t \, dt$$

$$v \Big|_0^v = 0.02e^t \Big|_0^t$$

$$v = [0.02(e^t - 1)] \text{ m/s} \quad (1)$$

Position:

$$\left(\pm \right) \quad ds = v \, dt$$

$$\int_0^s ds = \int_0^t 0.02(e^t - 1) \, dt$$

$$s \Big|_0^s = 0.02(e^t - t) \Big|_0^t$$

$$s = 0.02(e^t - t - 1) \text{ m}$$

When $s = 4 \text{ m}$,

$$4 = 0.02(e^t - t - 1)$$

$$e^t - t - 201 = 0$$

Solving the above equation by trial and error,

$$t = 5.329 \text{ s}$$

Thus, the velocity and acceleration when $s = 4 \text{ m}$ ($t = 5.329 \text{ s}$) are

$$v = 0.02(e^{5.329} - 1) = 4.11 \text{ m/s} \quad \textbf{Ans.}$$

$$a = 0.02e^{5.329} = 4.13 \text{ m/s}^2 \quad \textbf{Ans.}$$

Ans:

$$v = 4.11 \text{ m/s}$$

$$a = 4.13 \text{ m/s}^2$$

12–25.

Two particles A and B start from rest at the origin $s = 0$ and move along a straight line such that $a_A = (6t - 3) \text{ ft/s}^2$ and $a_B = (12t^2 - 8) \text{ ft/s}^2$, where t is in seconds. Determine the distance between them when $t = 4 \text{ s}$ and the total distance each has traveled in $t = 4 \text{ s}$.

SOLUTION

Velocity: The velocity of particles A and B can be determined using

$$dv_A = a_A dt$$

$$\int_0^{v_A} dv_A = \int_0^t (6t - 3) dt$$

$$v_A = 3t^2 - 3t$$

$$dv_B = a_B dt$$

$$\int_0^{v_B} dv_B = \int_0^t (12t^2 - 8) dt$$

$$v_B = 4t^3 - 8t$$

The times when particle A stops are

$$3t^2 - 3t = 0 \quad t = 0 \quad \text{and} \quad t = 1 \text{ s}$$

The times when particle B stops are

$$4t^3 - 8t = 0 \quad t = 0 \quad \text{and} \quad t = \sqrt{2} \text{ s}$$

Position: The position of particles A and B can be determined using

$$ds_A = v_A dt$$

$$\int_0^{s_A} ds_A = \int_0^t (3t^2 - 3t) dt$$

$$s_A = t^3 - \frac{3}{2} t^2$$

$$ds_B = v_B dt$$

$$\int_0^{s_B} ds_B = \int_0^t (4t^3 - 8t) dt$$

$$s_B = t^4 - 4t^2$$

The positions of particle A at $t = 1 \text{ s}$ and 4 s are

$$s_A|_{t=1\text{ s}} = 1^3 - \frac{3}{2}(1^2) = -0.500 \text{ ft}$$

$$s_A|_{t=4\text{ s}} = 4^3 - \frac{3}{2}(4^2) = 40.0 \text{ ft}$$

Particle A has traveled

$$d_A = 2(0.5) + 40.0 = 41.0 \text{ ft}$$

Ans.

The positions of particle B at $t = \sqrt{2} \text{ s}$ and 4 s are

$$s_B|_{t=\sqrt{2}\text{ s}} = (\sqrt{2})^4 - 4(\sqrt{2})^2 = -4 \text{ ft}$$

$$s_B|_{t=4\text{ s}} = (4)^4 - 4(4)^2 = 192 \text{ ft}$$

Particle B has traveled

$$d_B = 2(4) + 192 = 200 \text{ ft}$$

Ans.

At $t = 4 \text{ s}$ the distance between A and B is

$$\Delta s_{AB} = 192 - 40 = 152 \text{ ft}$$

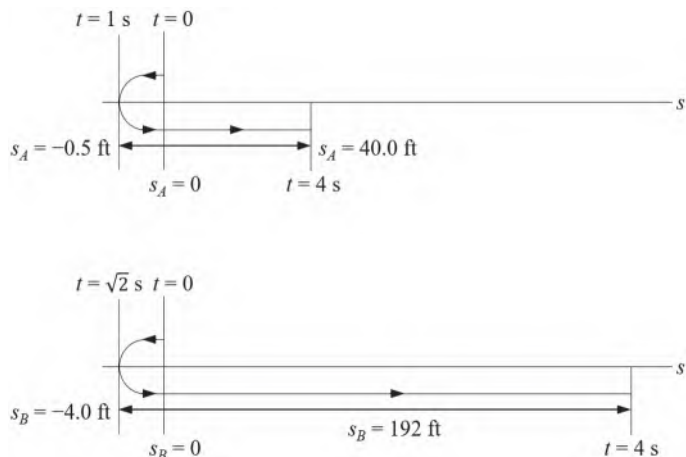
Ans.

Ans:

$$(s_T)_A = 41.0 \text{ ft}$$

$$(s_T)_B = 200 \text{ ft}$$

$$\Delta s_{AB} = 152 \text{ ft}$$



12-26.

The acceleration of a particle as it moves along a straight line is given by $a = (2t - 1) \text{ m/s}^2$, where t is in seconds. If $s = 1 \text{ m}$ and $v = 2 \text{ m/s}$ when $t = 0$, determine the particle's velocity and position when $t = 6 \text{ s}$. Also, determine the total distance the particle travels during this time period.

SOLUTION

$$\int_2^v dv = \int_0^t (2t - 1) dt$$

$$v = t^2 - t + 2$$

$$\int_1^s ds = \int_0^t (t^2 - t + 2) dt$$

$$s = \frac{1}{3}t^3 - \frac{1}{2}t^2 + 2t + 1$$

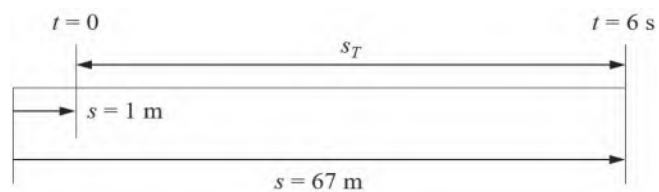
When $t = 6 \text{ s}$,

$$v = 32 \text{ m/s}$$

$$s = 67 \text{ m}$$

Since $v \neq 0$ then

$$s_T = 67 - 1 = 66 \text{ m}$$



Ans.

Ans.

Ans.

Ans:

$$v = 32 \text{ m/s}$$

$$s = 67 \text{ m}$$

$$s_T = 66 \text{ m}$$

12-27.

When a particle falls through the air, its initial acceleration $a = g$ diminishes until it is zero, and thereafter it falls at a constant or terminal velocity v_f . If this variation of the acceleration can be expressed as $a = (g/v_f^2)(v_f^2 - v^2)$, determine the time needed for the velocity to become $v = v_f/2$. Initially the particle falls from rest.

SOLUTION

$$\frac{dv}{dt} = a = \left(\frac{g}{v_f^2}\right)(v_f^2 - v^2)$$

$$\int_0^v \frac{dv}{v_f^2 - v^2} = \frac{g}{v_f^2} \int_0^t dt$$

$$\frac{1}{2v_f} \ln \left(\frac{v_f + v}{v_f - v} \right) \Big|_0^v = \frac{g}{v_f^2} t$$

$$t = \frac{v_f}{2g} \ln \left(\frac{v_f + v}{v_f - v} \right)$$

$$t = \frac{v_f}{2g} \ln \left(\frac{v_f + v_f/2}{v_f - v_f/2} \right)$$

$$t = 0.549 \left(\frac{v_f}{g} \right)$$

Ans.**Ans:**

$$t = 0.549 \left(\frac{v_f}{g} \right)$$

12–29.

Two cars A and B start from rest at a stop line. Car A has a constant acceleration of $a_A = 8 \text{ m/s}^2$, while car B has an acceleration of $a_B = (2t^{3/2}) \text{ m/s}^2$, where t is in seconds. Determine the distance between the cars when A reaches a velocity of $v_A = 120 \text{ km/h}$.

SOLUTION**For car A:**

$$(\rightarrow) \quad v^2 = v_0^2 + 2a_c(s - s_0)$$

$$\left[\frac{120(10)^3}{3600} \right]^2 = 0 + 2(8)(s_A - 0)$$

$$s_A = 69.44 \text{ m}$$

$$(\rightarrow) \quad v = v_0 + a_c t$$

$$\frac{120(10)^3}{3600} = 0 + 8t \quad t = 4.1667 \text{ s}$$

For car B:

$$(\rightarrow) \quad \frac{dv}{dt} = a$$

$$dv = 2t^{\frac{3}{2}} dt$$

$$\int_0^{v_B} dv = \int_0^t 2t^{\frac{3}{2}} dt$$

$$v_B = \frac{4}{5} t^{\frac{5}{2}}$$

$$(\rightarrow) \quad \frac{ds}{dt} = v$$

$$ds = \frac{4}{5} t^{\frac{5}{2}} dt$$

$$\int_0^{s_B} ds = \frac{4}{5} \int_0^{4.1667} t^{\frac{5}{2}} dt$$

$$s_B = 33.75 \text{ m}$$

The distance between cars A and B is $s_{AB} = 69.44 - 33.75 = 35.7 \text{ m}$

Ans.

Ans:
 $s_{AB} = 35.7 \text{ m}$

12–30.

A stone A is dropped from rest down a well, and in 1 s another stone B is dropped from rest. Determine the distance between the stones another second later.

SOLUTION

$$(+\downarrow) s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$s_A = 0 + 0 + \frac{1}{2}(32.2)(2)^2$$

$$s_A = 64.4 \text{ ft}$$

$$s_B = 0 + 0 + \frac{1}{2}(32.2)(1)^2$$

$$s_B = 16.1 \text{ ft}$$

$$\Delta s = 64.4 - 16.1 = 48.3 \text{ ft}$$

Ans.

Ans:
 $s_{AB} = 48.3 \text{ ft}$

12-31.

A particle is moving along a straight line such that its position is given by $s = (4t - t^2)$ ft, where t is in seconds. Determine the distance traveled from $t = 0$ to $t = 5$ s, the average velocity, and the average speed of the particle during this time interval.

SOLUTION

Total Distance Traveled: The velocity of the particle can be determined by applying

$$v = \frac{ds}{dt} = 4 - 2t$$

The time when the particle stops is

$$4 - 2t = 0 \quad t = 2 \text{ s}$$

The position of the particle at $t = 0, 2$ s and 5 s is

$$s|_{t=0} = 4(0) - (0^2) = 0$$

$$s|_{t=2 \text{ s}} = 4(2) - (2^2) = 4.00 \text{ ft}$$

$$s|_{t=5 \text{ s}} = 4(5) - (5^2) = -5.00 \text{ ft}$$

From the particle's path, the total distance is

$$s_T = 4.00 + 9.00 = 13.0 \text{ ft}$$

Ans.

Average Velocity: The displacement of the particle from $t = 0$ to 5 s is

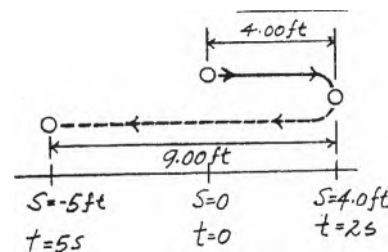
$$\Delta s = s|_{t=5 \text{ s}} - s|_{t=0} = -5.00 - 0 = -5.00 \text{ ft.}$$

$$v_{\text{avg}} = \frac{\Delta s}{\Delta t} = \frac{-5}{5} = -1.00 \text{ ft/s}$$

Ans.

Average Speed:

$$(v_{\text{avg}})_{\text{sp}} = \frac{s_T}{\Delta t} = \frac{13.0}{5} = 2.60 \text{ ft/s}$$

Ans.**Ans:**

$$s_T = 13.0 \text{ ft}$$

$$v_{\text{avg}} = -1.00 \text{ ft/s}$$

$$(v_{\text{avg}})_{\text{sp}} = 2.60 \text{ ft/s}$$

***12–32.**

A particle has an initial speed of 27 m/s. If it experiences a deceleration of $a = (-6t) \text{ m/s}^2$ where t is in seconds, determine its velocity after it has traveled 10 m. How much time does this take?

SOLUTION

Velocity:

$$\begin{aligned}(\pm) \quad dv &= a \, dt \\ \int_{27}^v dv &= \int_0^t (-6t) dt \\ v \Big|_{27}^v &= (-3t^2) \Big|_0^t \\ v &= (27 - 3t^2) \text{ m/s}\end{aligned}$$

$$\begin{aligned}(\pm) \quad ds &= v \, dt \\ \int_0^s ds &= \int_0^t (27 - 3t^2) dt \\ s \Big|_0^s &= (27t - t^3) \Big|_0^t \\ s &= (27t - t^3) \text{ m/s}\end{aligned}$$

When $s = 10 \text{ m}$,

$$t = 0.372 \text{ s}$$

Ans.

$$v = 26.6 \text{ m/s}$$

Ans.

Ans:
 $t = 0.372 \text{ s}$
 $v = 26.6 \text{ m/s}$

12–33.

As a body is projected to a high altitude above the earth's surface, the variation of the acceleration of gravity with respect to altitude y must be taken into account. Neglecting air resistance, this acceleration is determined from the formula $a = -g_0[R^2/(R + y)^2]$, where g_0 is the constant gravitational acceleration at sea level, R is the radius of the earth, and the positive direction is measured upward. If $g_0 = 9.81 \text{ m/s}^2$ and $R = 6356 \text{ km}$, determine the minimum initial velocity (escape velocity) at which a projectile should be shot vertically from the earth's surface so that it does not fall back to the earth. *Hint:* This requires that $v = 0$ as $y \rightarrow \infty$.

SOLUTION

$$v \, dv = a \, dy$$

$$\int_v^0 v \, dv = -g_0 R^2 \int_0^\infty \frac{dy}{(R + y)^2}$$

$$\left. \frac{v^2}{2} \right|_v^0 = \left. \frac{g_0 R^2}{R + y} \right|_0^\infty$$

$$v = \sqrt{2g_0 R}$$

$$= \sqrt{2(9.81)(6356)(10)^3}$$

$$= 11167 \text{ m/s} = 11.2 \text{ km/s}$$

Ans.

Ans:
 $v_e = 11.2 \text{ km/s}$

12–34.

Accounting for the variation of gravitational acceleration a with respect to altitude y (see Prob. 12–33), derive an equation that relates the velocity of a freely falling particle to its altitude. Assume that the particle is released from rest at an altitude y_0 from the earth's surface. With what velocity does the particle strike the earth if it is released from rest at an altitude $y_0 = 500$ km? Use the numerical data in Prob. 12–33.

SOLUTION

From Prob. 12–33,

$$(+\uparrow) \quad a = -g_0 \frac{R^2}{(R + y)^2}$$

Since $a \, dy = v \, dv$

then

$$-g_0 R^2 \int_{y_0}^y \frac{dy}{(R + y)^2} = \int_0^v v \, dv$$

$$g_0 R^2 \left[\frac{1}{R + y} \right]_{y_0}^y = \frac{v^2}{2}$$

$$g_0 R^2 \left[\frac{1}{R + y} - \frac{1}{R + y_0} \right] = \frac{v^2}{2}$$

Thus

$$v = -R \sqrt{\frac{2g_0(y_0 - y)}{(R + y)(R + y_0)}}$$

Ans.

When $y_0 = 500$ km, $y = 0$,

$$v = -6356(10^3) \sqrt{\frac{2(9.81)(500)(10^3)}{6356(6356 + 500)(10^6)}}$$

$$v = -3016 \text{ m/s} = 3.02 \text{ km/s} \downarrow$$

Ans.

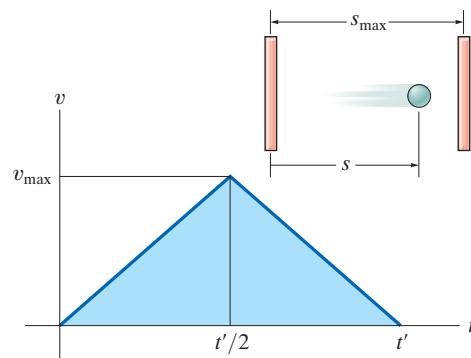
Ans:

$$v = -R \sqrt{\frac{2g_0(y_0 - y)}{(R + y)(R + y_0)}}$$

$$v_{\text{imp}} = 3.02 \text{ km/s}$$

12–35.

The $v-t$ graph for a particle moving through an electric field from one plate to another has the shape shown in the figure. The acceleration and deceleration that occur are constant and both have a magnitude of 4 m/s^2 . If the plates are spaced 200 mm apart, determine the maximum velocity $v_{\max}$ and the time t' for the particle to travel from one plate to the other. Also draw the $s-t$ graph. When $t = t'/2$ the particle is at $s = 100 \text{ mm}$.



SOLUTION

$$a_c = 4 \text{ m/s}^2$$

$$\frac{s_{\max}}{2} = 100 \text{ mm} = 0.1 \text{ m}$$

$$v^2 = v_0^2 + 2 a_c (s - s_0)$$

$$v_{\max}^2 = 0 + 2(4)(0.1 - 0)$$

$$v_{\max} = 0.89443 \text{ m/s} = 0.894 \text{ m/s}$$

$$v = v_0 + a_c t$$

$$0.89443 = 0 + 4\left(\frac{t'}{2}\right)$$

$$t' = 0.44721 \text{ s} = 0.447 \text{ s}$$

$$s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$s = 0 + 0 + \frac{1}{2} (4)(t)^2$$

$$s = (2t^2) \text{ m}$$

$$\text{When } t = \frac{0.44721}{2} = 0.2236 = 0.224 \text{ s,}$$

$$s = 0.1 \text{ m}$$

$$\int_{0.8944}^v dv = - \int_{0.2236}^t 4 dt$$

$$v = -4t + 1.789$$

$$\int_{0.1}^s ds = \int_{0.2236}^t (-4t + 1.789) dt$$

$$s = (-2t^2 + 1.789t - 0.2) \text{ m}$$

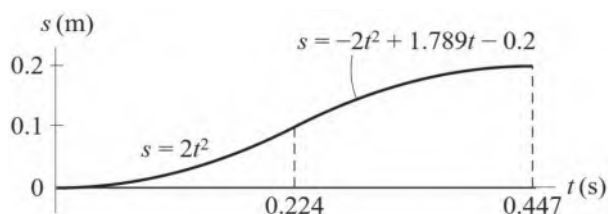
$$\text{When } t = 0.447 \text{ s,}$$

$$s = 0.2 \text{ m}$$

Ans.

Ans.

Ans.



Ans.

Ans:

$$v_{\max} = 0.894 \text{ m/s}$$

$$t' = 0.447 \text{ s}$$

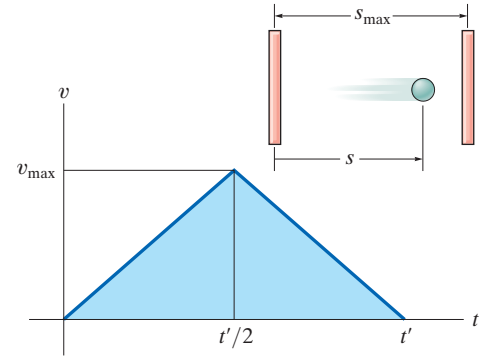
$$\text{For } 0 \leq t < 0.224 \text{ s, } s = (2t^2) \text{ m;}$$

$$\text{For } 0.224 \text{ s} < t \leq 0.447 \text{ s,}$$

$$s = (-2t^2 + 1.79t - 0.2) \text{ m}$$

***12–36.**

The $v-t$ graph for a particle moving through an electric field from one plate to another has the shape shown in the figure, where $t' = 0.2$ s and $v_{\max} = 10$ m/s. Draw the $s-t$ and $a-t$ graphs for the particle. When $t = t'/2$ the particle is at $s = 0.5$ m.



SOLUTION

For $0 \leq t < 0.1$ s,

$$a = \frac{dv}{dt} = 100$$

Ans.

$$v = 100t$$

$$ds = v dt$$

$$\int_0^s ds = \int_0^t 100t dt$$

$$s = 50t^2$$

Ans.

When $t = 0.1$ s,

$$s = 0.5 \text{ m}$$

For $0.1 \text{ s} < t \leq 0.2$ s,

$$a = \frac{dv}{dt} = -100$$

Ans.

$$v = -100t + 20$$

$$ds = v dt$$

$$\int_{0.5}^s ds = \int_{0.1}^t (-100t + 20) dt$$

$$s - 0.5 = (-50t^2 + 20t - 1.5)$$

$$s = -50t^2 + 20t - 1$$

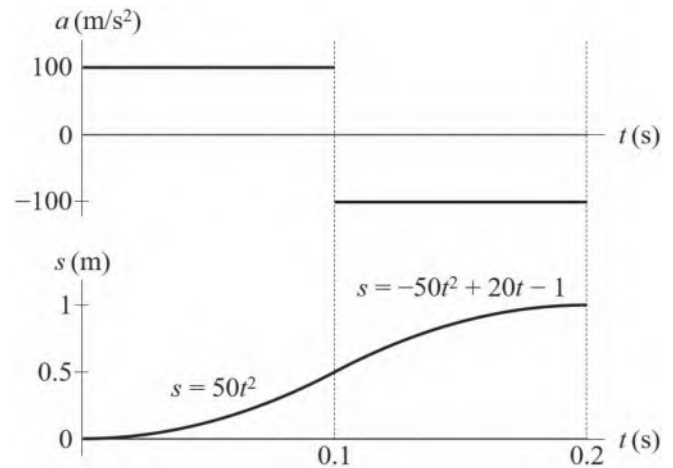
Ans.

When $t = 0.2$ s,

$$s = 1 \text{ m}$$

When $t = 0.1$ s, $s = 0.5$ m and a changes from 100 m/s^2

to -100 m/s^2 . When $t = 0.2$ s, $s = 1$ m.



Ans:

For $0 \leq t < 0.1$ s, $a = 100 \text{ m/s}^2$, $s = (50t^2) \text{ m}$

For $0.1 \text{ s} < t \leq 0.2$ s, $a = -100 \text{ m/s}^2$, $s = (-50t^2 + 20t - 1) \text{ m}$

12–37.

Two rockets start from rest at the same elevation. Rocket *A* accelerates vertically at 20 m/s^2 for 12 s and then maintains a constant speed. Rocket *B* accelerates at 15 m/s^2 until reaching a constant speed of 150 m/s . Construct the a - t , v - t , and s - t graphs for each rocket until $t = 20 \text{ s}$. What is the distance between the rockets when $t = 20 \text{ s}$?

SOLUTION

For Rocket A:

For $t < 12 \text{ s}$

$$(+\uparrow) v_A = (v_A)_0 + a_A t$$

$$v_A = 0 + 20 t$$

$$v_A = 20 t$$

Ans.

$$(+\uparrow) s_A = (s_A)_0 + (v_A)_0 t + \frac{1}{2} a_A t^2$$

$$s_A = 0 + 0 + \frac{1}{2} (20) t^2$$

$$s_A = 10 t^2$$

Ans.

When $t = 12 \text{ s}$, $v_A = 240 \text{ m/s}$

$$s_A = 1440 \text{ m}$$

For $t > 12 \text{ s}$

$$v_A = 240 \text{ m/s}$$

Ans.

$$s_A = 1440 + 240(t - 12) = 240(t - 6) \text{ m}$$

Ans.

For Rocket B:

For $t < 10 \text{ s}$

$$(+\uparrow) v_B = (v_B)_0 + a_B t$$

$$v_B = 0 + 15 t$$

$$v_B = 15 t$$

Ans.

$$(+\uparrow) s_B = (s_B)_0 + (v_B)_0 t + \frac{1}{2} a_B t^2$$

$$s_B = 0 + 0 + \frac{1}{2} (15) t^2$$

$$s_B = 7.5 t^2$$

Ans.

When $t = 10 \text{ s}$, $v_B = 150 \text{ m/s}$

$$s_B = 750 \text{ m}$$

For $t > 10 \text{ s}$

$$v_B = 150 \text{ m/s}$$

Ans.

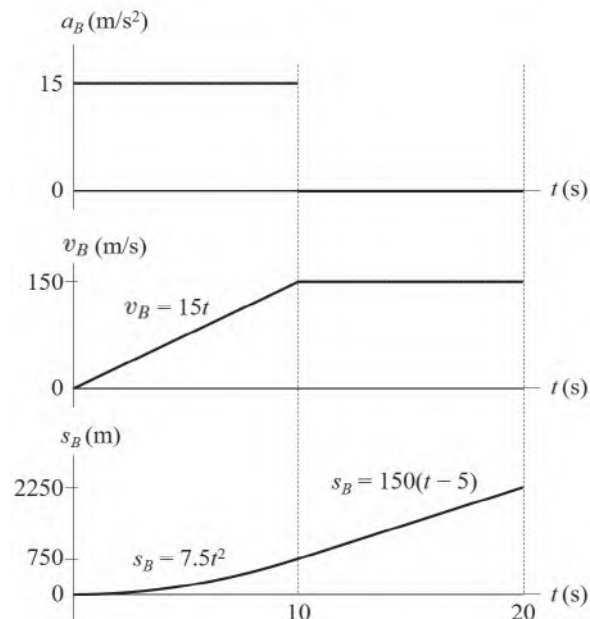
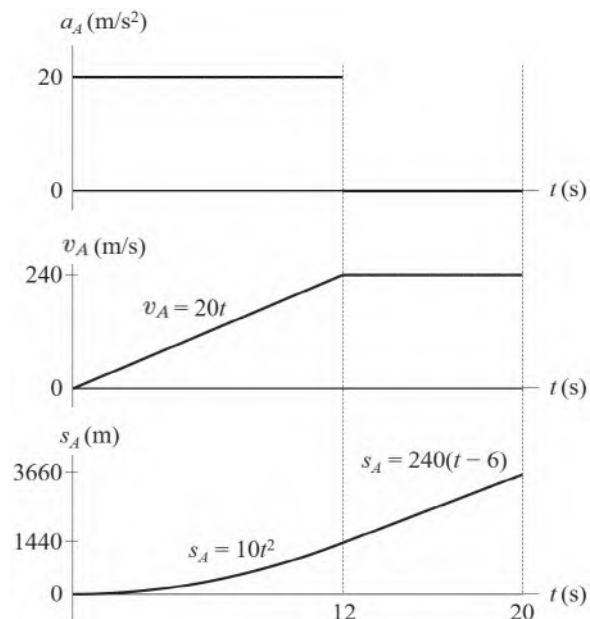
$$s_B = 750 + 150(t - 10) = 150(t - 5) \text{ m}$$

Ans.

When $t = 20 \text{ s}$, $s_A = 3360 \text{ m}$, $s_B = 2250 \text{ m}$

$$\Delta s = 1110 \text{ m} = 1.11 \text{ km}$$

Ans.



Ans:

For $0 \leq t < 12 \text{ s}$, $a_A = 20 \text{ m/s}^2$, $v_A = (20t) \text{ m/s}$, $s_A = (10t^2) \text{ m}$

For $12 \text{ s} < t \leq 20 \text{ s}$, $a_A = 0$, $v_A = 240 \text{ m/s}$, $s_A = 240(t - 6) \text{ m}$

For $0 \leq t < 10 \text{ s}$, $a_B = 15 \text{ m/s}^2$, $v_B = (15t) \text{ m/s}$, $s_B = (7.5t^2) \text{ m}$

For $10 \text{ s} < t \leq 20 \text{ s}$, $a_B = 0$, $v_B = 150 \text{ m/s}$, $s_B = 150(t - 5) \text{ m}$

$s_{AB} = 1.11 \text{ km}$

12-38.

A particle starts from $s = 0$ and travels along a straight line with a velocity $v = (t^2 - 4t + 3)$ m/s, where t is in seconds. Construct the $v-t$ and $a-t$ graphs for the time interval $0 \leq t \leq 4$ s.

SOLUTION

$a-t$ Graph:

$$a = \frac{dv}{dt} = \frac{d}{dt}(t^2 - 4t + 3)$$

$$a = (2t - 4) \text{ m/s}^2$$

Ans.

Thus,

$$a|_{t=0} = 2(0) - 4 = -4 \text{ m/s}^2$$

$$a|_{t=2\text{ s}} = 0$$

$$a|_{t=4\text{ s}} = 2(4) - 4 = 4 \text{ m/s}^2$$

The $a-t$ graph is shown in Fig. (a).

$v-t$ Graph: The slope of the $v-t$ graph is zero when $a = \frac{dv}{dt} = 0$. Thus,

$$a = 2t - 4 = 0 \quad t = 2 \text{ s}$$

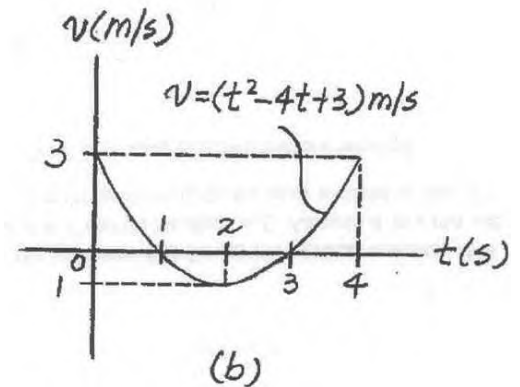
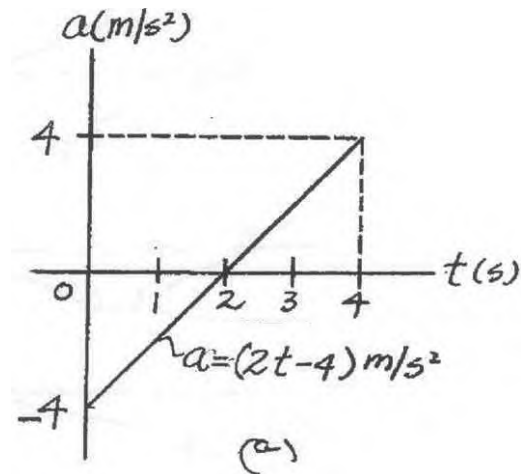
The velocity of the particle at $t = 0, 2$ s, and 4 s is

$$v|_{t=0} = 0^2 - 4(0) + 3 = 3 \text{ m/s}$$

$$v|_{t=2\text{ s}} = 2^2 - 4(2) + 3 = -1 \text{ m/s}$$

$$v|_{t=4\text{ s}} = 4^2 - 4(4) + 3 = 3 \text{ m/s}$$

The $v-t$ graph is shown in Fig. (b).



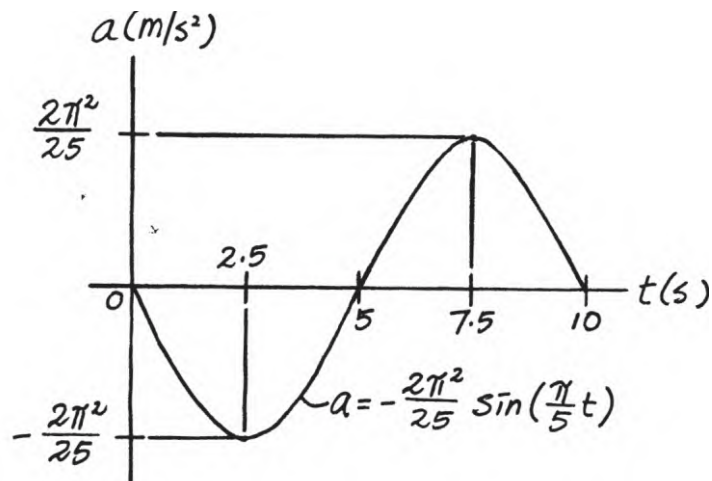
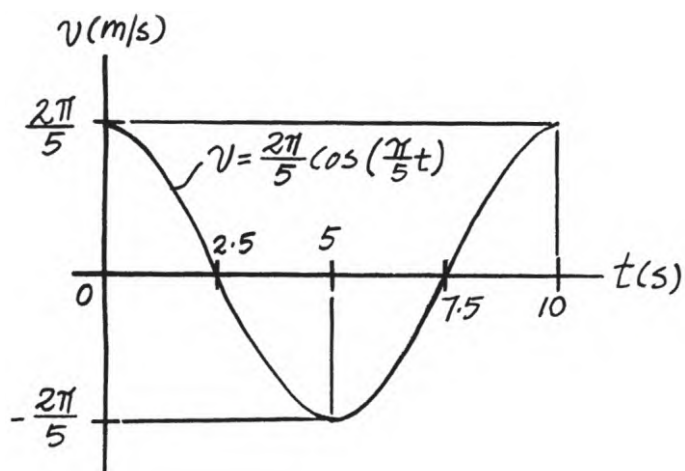
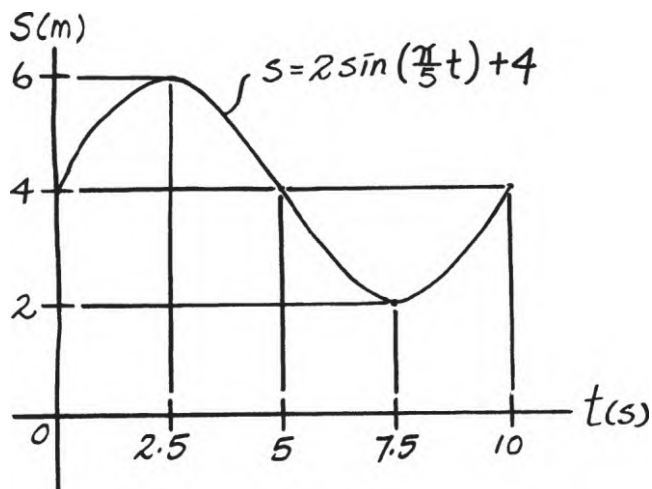
Ans:

$$a = (2t - 4) \text{ m/s}^2$$

12-39.

If the position of a particle is defined by $s = [2 \sin(\pi t/5) + 4]$ m, where t is in seconds, construct the s - t , v - t , and a - t graphs for $0 \leq t \leq 10$ s.

SOLUTION



Ans:

$$v = \frac{2\pi}{5} \cos\left(\frac{\pi}{5}t\right) \text{ m/s}$$

$$a = -\frac{2\pi^2}{25} \sin\left(\frac{\pi}{5}t\right) \text{ m/s}^2$$

***12–40.**

An airplane starts from rest, travels 5000 ft down a runway, and after uniform acceleration takes off with a speed of 162 mi/h. It then climbs in a straight line with a uniform acceleration of 3 ft/s² until it reaches a constant speed of 220 mi/h. Draw the s - t , v - t , and a - t graphs that describe the motion.

SOLUTION

$$v_1 = 0$$

$$v_2 = 162 \frac{\text{mi}}{\text{h}} \frac{(1 \text{ h}) (5280 \text{ ft})}{(3600 \text{ s})(1 \text{ mi})} = 237.6 \text{ ft/s}$$

$$v_2^2 = v_1^2 + 2 a_c (s_2 - s_1)$$

$$(237.6)^2 = 0^2 + 2(a_c)(5000 - 0)$$

$$a_c = 5.64538 \text{ ft/s}^2$$

$$v_2 = v_1 + a_c t$$

$$237.6 = 0 + 5.64538 t$$

$$t = 42.09 = 42.1 \text{ s}$$

$$v_3 = 220 \frac{\text{mi}}{\text{h}} \frac{(1 \text{ h}) (5280 \text{ ft})}{(3600 \text{ s})(1 \text{ mi})} = 322.67 \text{ ft/s}$$

$$v_3^2 = v_2^2 + 2 a_c (s_3 - s_2)$$

$$(322.67)^2 = (237.6)^2 + 2(3)(s - 5000)$$

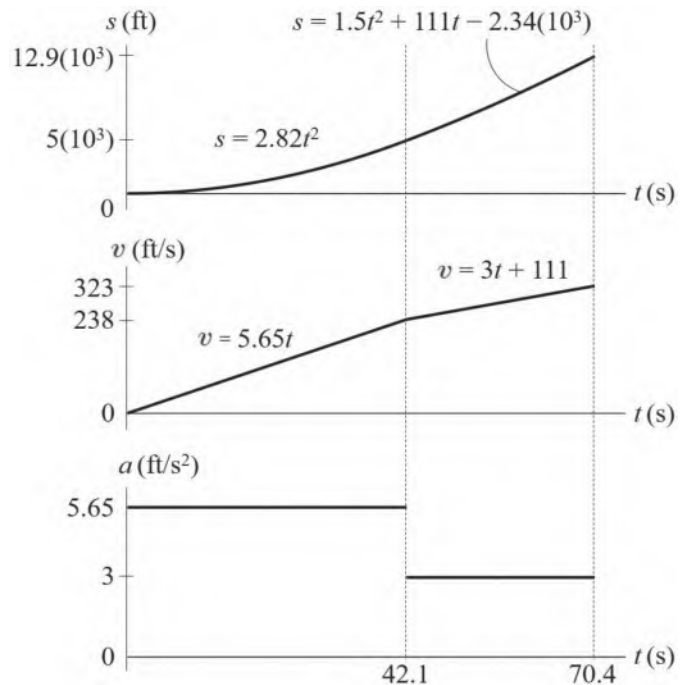
$$s = 12\,943.34 \text{ ft}$$

$$v_3 = v_2 + a_c t$$

$$322.67 = 237.6 + 3 t$$

$$t = 28.4 \text{ s}$$

Ans.



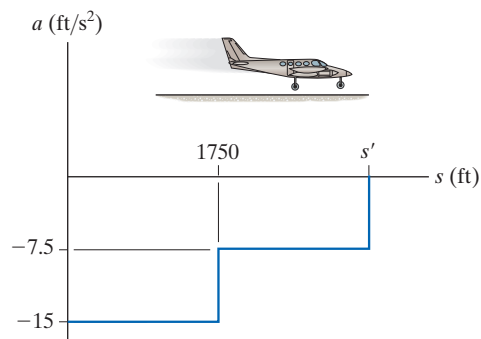
Ans:

For $0 \leq t < 42.1 \text{ s}$, $a = 5.65 \text{ ft/s}^2$, $v = (5.65t) \text{ ft/s}$, $s = (2.82t^2) \text{ ft}$

For $42.1 \text{ s} < t \leq 70.4 \text{ s}$, $a = 3 \text{ ft/s}^2$, $v = (3t + 111) \text{ ft/s}$, $s = [1.5t^2 + 111t - 2.34(10^3)] \text{ ft}$

12-41.

The airplane lands at 250 ft/s on a straight runway and has a deceleration described by the graph. Determine the distance s' traveled before its speed is decreased to 25 ft/s. Draw the v - s graph.



SOLUTION

v - s Graph: For $0 \leq s < 1750$ ft, the initial condition is $v = 250$ ft/s at $s = 0$.

$$(\pm) \quad v dv = a ds$$

$$\int_{250 \text{ ft/s}}^v v dv = \int_0^s -15 ds$$

$$\frac{v^2}{2} \Big|_{250 \text{ ft/s}}^v = -15s \Big|_0^s$$

$$v = (\sqrt{62\,500 - 30s}) \text{ ft/s}$$

At $s = 1750$ ft,

$$v|_{s=1750 \text{ ft}} = \sqrt{62\,500 - 30(1750)} = 100 \text{ ft/s}$$

For $1750 \text{ ft} < s \leq s'$, the initial condition is $v = 100$ ft/s at $s = 1750$ ft.

$$(\pm) \quad v dv = a ds$$

$$\int_{100 \text{ ft/s}}^v v dv = \int_{1750 \text{ ft}}^s -7.5 ds$$

$$\frac{v^2}{2} \Big|_{100 \text{ ft/s}}^v = (-7.5s) \Big|_{1750 \text{ ft}}^s$$

$$v = \sqrt{36\,250 - 15s}$$

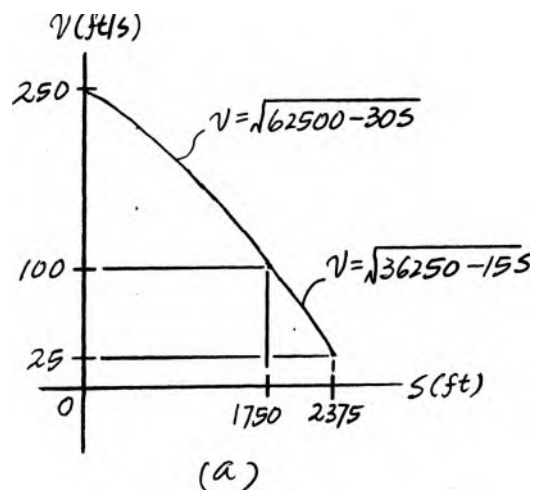
When $v = 25$ ft/s

$$25 = \sqrt{36\,250 - 15s}$$

$$s' = 2375 \text{ ft}$$

The v - s graph is shown in Fig. (a).

Ans.



Ans.

Ans.

Ans:

$$s' = 2.38(10^3) \text{ ft}$$

For $0 \leq s < 1750$ ft,

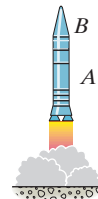
$$v = \sqrt{62.5(10^3) - 30s} \text{ ft/s}$$

For $1750 \text{ ft} < s \leq 2375$ ft,

$$v = \sqrt{36.2(10^3) - 15s} \text{ ft/s}$$

12-42.

A two-stage missile is fired vertically from rest with the acceleration shown. In 15 s the first stage *A* burns out and the second stage *B* ignites. Plot the v - t and s - t graphs which describe the two-stage motion of the missile for $0 \leq t \leq 20$ s.



SOLUTION

Since $v = \int a \, dt$, the constant lines of the a - t graph become sloping lines for the v - t graph.

The numerical values for each point are calculated from the total area under the a - t graph to the point.

$$\text{At } t = 15 \text{ s, } v = (18)(15) = 270 \text{ m/s}$$

$$\text{At } t = 20 \text{ s, } v = 270 + (25)(20 - 15) = 395 \text{ m/s}$$

Since $s = \int v \, dt$, the sloping lines of the v - t graph become parabolic curves for the s - t graph.

The numerical values for each point are calculated from the total area under the v - t graph to the point.

$$\text{At } t = 15 \text{ s, } s = \frac{1}{2}(15)(270) = 2025 \text{ m}$$

$$\text{At } t = 20 \text{ s, } s = 2025 + 270(20 - 15) + \frac{1}{2}(395 - 270)(20 - 15) = 3687.5 \text{ m} = 3.69 \text{ km}$$

Also:

$$0 \leq t < 15 \text{ s:}$$

$$a = 18$$

$$v = v_0 + a_c t = 0 + 18t$$

$$s = s_0 + v_0 t + \frac{1}{2} a_c t^2 = 0 + 0 + 9t^2$$

$$\text{At } t = 15 \text{ s:}$$

$$v = 18(15) = 270$$

$$s = 9(15)^2 = 2025$$

$$15 \text{ s} < t \leq 20 \text{ s:}$$

$$a = 25$$

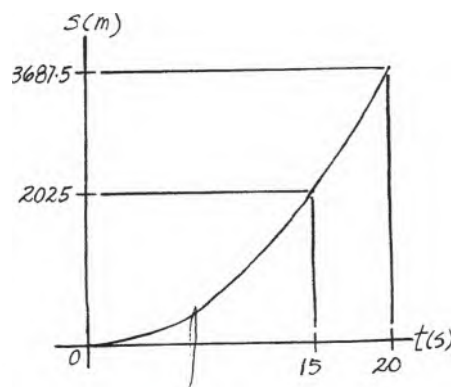
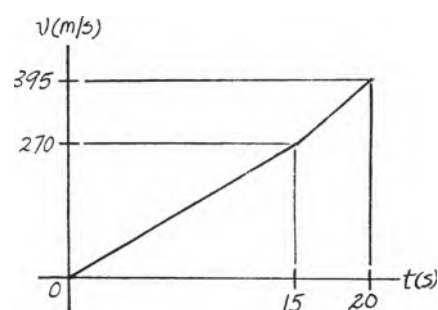
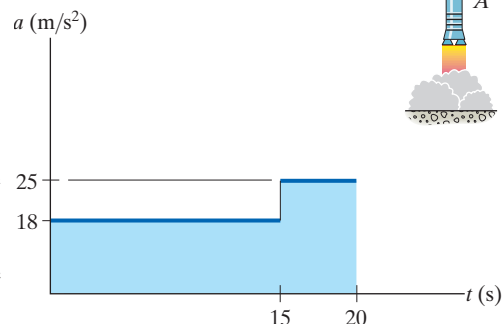
$$v = v_0 + a_c t = 270 + 25(t - 15)$$

$$s = s_0 + v_0 t + \frac{1}{2} a_c t^2 = 2025 + 270(t - 15) + \frac{1}{2}(25)(t - 15)^2$$

$$\text{When } t = 20 \text{ s:}$$

$$v = 395 \text{ m/s}$$

$$s = 3687.5 \text{ m} = 3.69 \text{ km}$$



Ans.

Ans.

Ans:

For $0 \leq t < 15$ s,

$$v = (18t) \text{ m/s}$$

$$s = (9t^2) \text{ m}$$

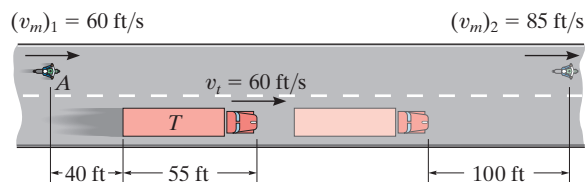
For $15 \text{ s} < t \leq 20$ s,

$$v = (25t - 105) \text{ m/s}$$

$$s = (12.5t^2 - 105t + 788) \text{ m}$$

12-43.

A motorcyclist at *A* is traveling at 60 ft/s when he wishes to pass the truck *T* which is traveling at a constant speed of 60 ft/s. To do so the motorcyclist accelerates at 6 ft/s² until reaching a maximum speed of 85 ft/s. If he then maintains this speed, determine the time needed for him to reach a point located 100 ft in front of the truck. Draw the *v*-*t* and *s*-*t* graphs for the motorcycle during this time.



SOLUTION

Motorcycle:

Time to reach 85 ft/s,

$$v = v_0 + a_c t$$

$$85 = 60 + 6t$$

$$t = 4.167 \text{ s}$$

Distance traveled,

$$v^2 = v_0^2 + 2a_c(s - s_0)$$

$$(85)^2 = (60)^2 + 2(6)(s_m - 0)$$

$$s_m = 302.08 \text{ ft}$$

In $t = 4.167$ s, truck travels

$$s_t = 60(4.167) = 250 \text{ ft}$$

Further distance for motorcycle to travel: $40 + 55 + 250 + 100 - 302.08 = 142.92$ ft

Motorcycle:

$$s = s_0 + v_0 t$$

$$(s + 142.92) = 0 + 85t'$$

Truck:

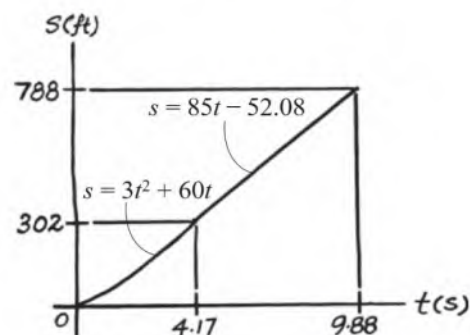
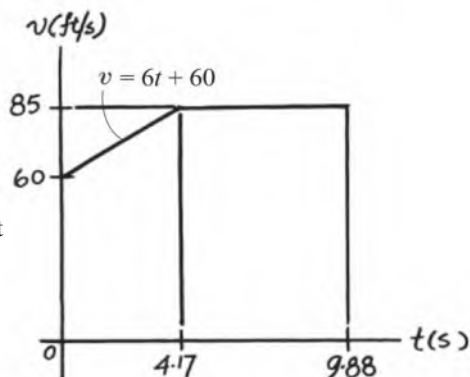
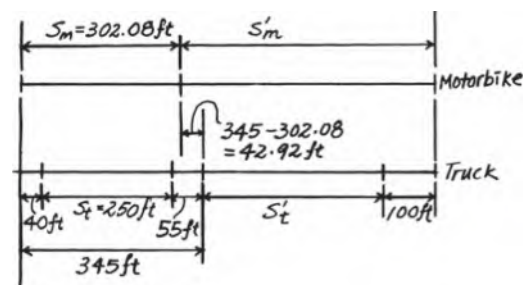
$$s = 0 + 60t'$$

Thus $t' = 5.717$ s

$$t = 4.167 + 5.717 = 9.88 \text{ s}$$

Total distance motorcycle travels

$$s_T = 302.08 + 85(5.717) = 788 \text{ ft}$$



Ans.

Ans:

$$t = 9.88 \text{ s}$$

For $0 \leq t < 4.17$ s,

$$v = (6t + 60) \text{ ft/s}$$

$$s = (3t^2 + 60t) \text{ ft}$$

For $4.17 \text{ s} < t \leq 9.88 \text{ s}$,

$$v = 85 \text{ ft/s}$$

$$s = (85t - 52.1) \text{ ft}$$

***12–44.**

The $v-t$ graph for the motion of a car as it moves along a straight road is shown. Draw the $a-t$ graph and determine the maximum acceleration during the 30-s time interval. The car starts from rest at $s = 0$.

SOLUTION

For $0 \leq t \leq 10$ s:

$$v = 0.4t^2$$

$$a = \frac{dv}{dt} = 0.8t$$

At $t = 10$ s:

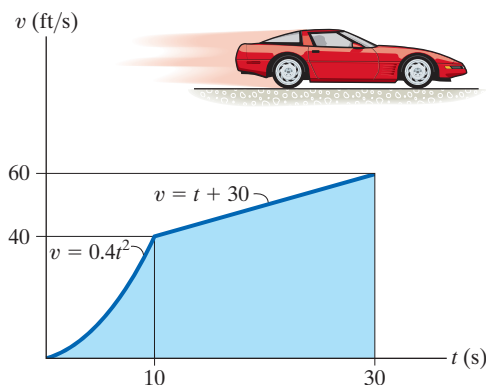
$$a = 8 \text{ ft/s}^2$$

For $10 \text{ s} < t \leq 30 \text{ s}$:

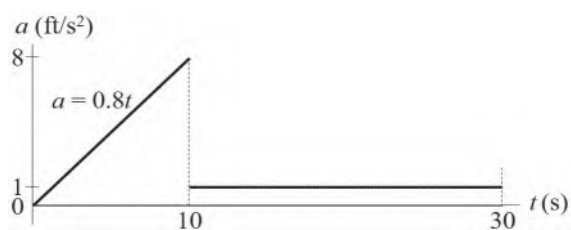
$$v = t + 30$$

$$a = \frac{dv}{dt} = 1$$

$$a_{\max} = 8 \text{ ft/s}^2$$



Ans.



Ans.

Ans.

Ans:

$$a_{\max} = 8 \text{ ft/s}^2$$

For $0 \leq t < 10$ s,

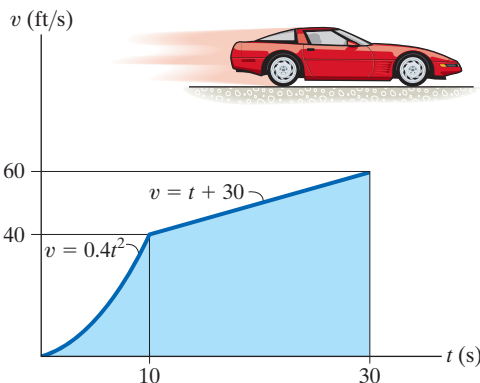
$$a = (0.8t) \text{ ft/s}^2$$

For $10 \text{ s} < t \leq 30 \text{ s}$,

$$a = 1 \text{ ft/s}^2$$

12–45.

The $v-t$ graph for the motion of a car as it moves along a straight road is shown. Draw the $s-t$ graph and determine the average speed and the distance traveled for the 30-s time interval. The car starts from rest at $s = 0$.



SOLUTION

For $0 \leq t \leq 10$ s,

$$v = 0.4t^2$$

$$ds = v dt$$

$$\int_0^s ds = \int_0^t 0.4t^2 dt$$

$$s = 0.1333t^3$$

At $t = 10$ s,

$$s = 133.3 \text{ ft}$$

For $10 \text{ s} < t \leq 30 \text{ s}$,

$$v = t + 30$$

$$ds = v dt$$

$$\int_{133.3}^s ds = \int_{10}^t (t + 30) dt$$

$$s = 0.5t^2 + 30t - 216.7$$

At $t = 30$ s,

$$s = 1133 \text{ ft}$$

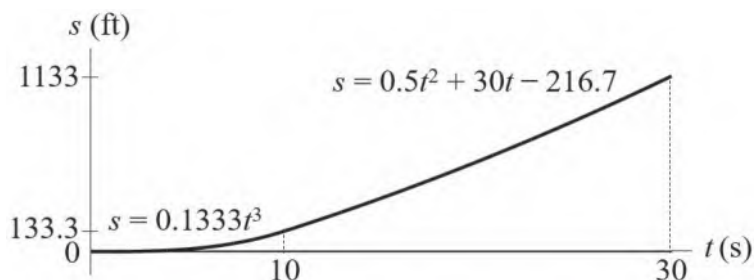
$$(v_{\text{avg}})_{\text{sp}} = \frac{s_T}{\Delta t} = \frac{1133}{30} = 37.8 \text{ ft/s}$$

$$s_T = 1133 \text{ ft} = 1.13(10^3) \text{ ft}$$

When $t = 10$ s, $s = 133 \text{ ft}$.

When $t = 30$ s, $s = s_T = 1.13(10^3) \text{ ft}$

Ans.



Ans.

Ans.

Ans.

Ans:

For $0 \leq t < 10$ s,

$$s = (0.133t^3) \text{ ft}$$

For $10 \text{ s} < t \leq 30 \text{ s}$,

$$s = (0.5t^2 + 30t - 217) \text{ ft}$$

$$(v_{\text{avg}})_{\text{sp}} = 37.8 \text{ ft/s}$$

$$s_T = 1.13(10^3) \text{ ft}$$

12-46.

The $a-s$ graph for a train traveling along a straight track is given for the first 400 m of its motion. Plot the $v-s$ graph. $v = 0$ at $s = 0$.

SOLUTION

$$0 \leq s < 200 \text{ m:} \quad a = \frac{1}{100}s$$

$$a \, ds = v \, dv$$

$$\int_0^s \frac{1}{100}s \, ds = \int_0^v v \, dv$$

$$\frac{1}{200}s^2 = \frac{1}{2}v^2$$

$$v = 0.1s$$

$$\text{At } s = 200 \text{ m,} \quad v = 20 \text{ m/s}$$

$$200 \text{ m} < s \leq 400 \text{ m:} \quad a = 2$$

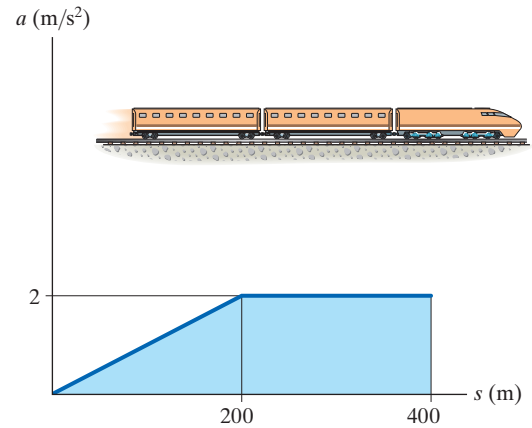
$$a \, ds = v \, dv$$

$$\int_{200}^s 2 \, ds = \int_{20}^v v \, dv$$

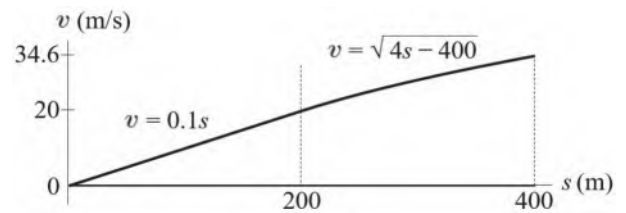
$$2(s - 200) = \frac{1}{2}(v^2 - 400)$$

$$v^2 = 4s - 400$$

$$\text{At } s = 400 \text{ m,} \quad v = \sqrt{4(400) - 400} = 34.6 \text{ m/s}$$



Ans.



Ans:

For $0 \leq s < 200 \text{ m}$,

$v = (0.1s) \text{ m/s}$

For $200 \text{ m} < s \leq 400 \text{ m}$,

$v = \sqrt{4s - 400} \text{ m/s}$

12–47.

A missile starting from rest travels along a straight track and for 10 s has an acceleration as shown. Draw the $v-t$ graph that describes the motion and find the distance traveled in 10 s.

SOLUTION

For $0 < t \leq 5$ s,

$$a = 6t$$

$$dv = a dt$$

$$\int_0^v dv = \int_0^t 6t dt$$

$$v = 3t^2$$

When $t = 5$ s,

$$v = 75 \text{ m/s}$$

For $5 \text{ s} < t \leq 10 \text{ s}$,

$$a = 2t + 20$$

$$dv = a dt$$

$$\int_{75}^v dv = \int_5^t (2t + 20) dt$$

$$v - 75 = t^2 + 20t - 125$$

$$v = t^2 + 20t - 50$$

When $t = 10$ s,

$$v = 250 \text{ m/s}$$

Distance at $t = 5$ s:

$$ds = v dt$$

$$\int_0^s ds = \int_0^5 3t^2 dt$$

$$s = (5)^3 = 125 \text{ m}$$

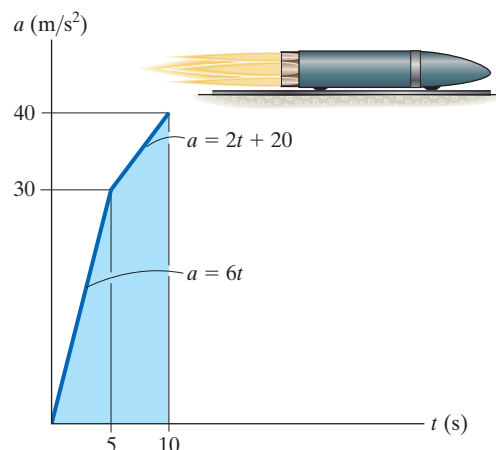
Distance at $t = 10$ s:

$$ds = v dt$$

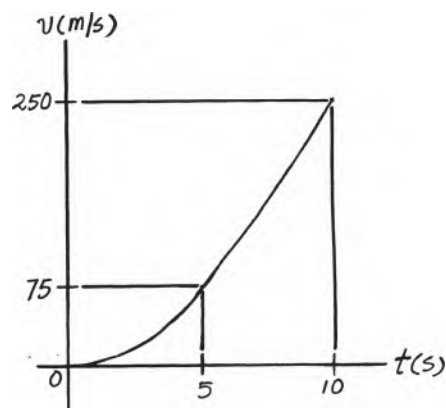
$$\int_{125}^s ds = \int_5^{10} (t^2 + 20t - 50) dt$$

$$s - 125 = \left. \frac{1}{3}t^3 + 10t^2 - 50t \right|_5^{10}$$

$$s = 917 \text{ m}$$



Ans.



Ans.

Ans:

$$s_T = 917 \text{ m}$$

For $0 \leq t < 5$ s,

$$v = (3t^2) \text{ m/s}$$

For $5 \text{ s} < t \leq 10 \text{ s}$,

$$v = (t^2 + 20t - 50) \text{ m/s}$$

Ans.

***12–48.**

The position of a cyclist traveling along a straight road is described by the graph. Construct the $v-t$ and $a-t$ graphs.



SOLUTION

$v-t$ Graph: For the time interval $0 \leq t < 10$ s,

$$\left(\pm \right) \quad v = \frac{ds}{dt} = \frac{d}{dt} (0.05t^3) = (0.15t^2) \text{ m/s}$$

When $t = 0$ and 10 s,

$$v|_{t=0} = 0.15(0^2) = 0 \quad v|_{t=10 \text{ s}} = 0.15(10^2) = 15 \text{ m/s}$$

For the time interval $10 \text{ s} < t \leq 20$ s,

$$\left(\pm \right) \quad v = \frac{ds}{dt} = \frac{d}{dt} (-0.625t^2 + 27.5t - 162.5) = (-1.25t + 27.5) \text{ m/s} \quad \text{Ans.}$$

When $t = 10$ s and 20 s,

$$v|_{t=10 \text{ s}} = -1.25(10) + 27.5 = 15 \text{ m/s}$$

$$v|_{t=20 \text{ s}} = -1.25(20) + 27.5 = 2.5 \text{ m/s}$$

The $v-t$ graph is shown in Fig. (a).

$a-t$ Graph: For the time interval $0 \leq t < 10$ s,

$$\left(\pm \right) \quad a = \frac{dv}{dt} = \frac{d}{dt} (0.15t^2) = (0.3t) \text{ m/s}^2 \quad \text{Ans.}$$

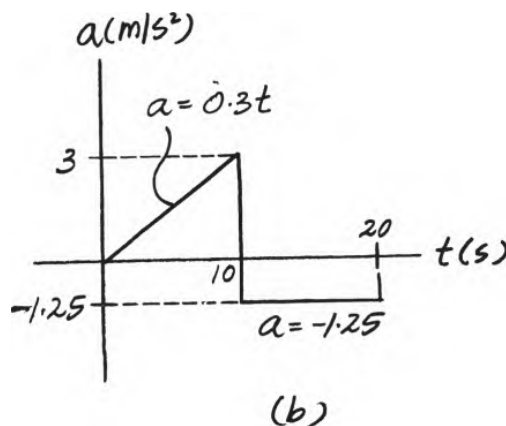
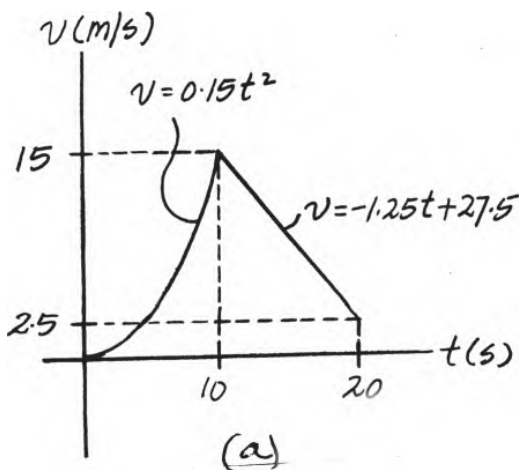
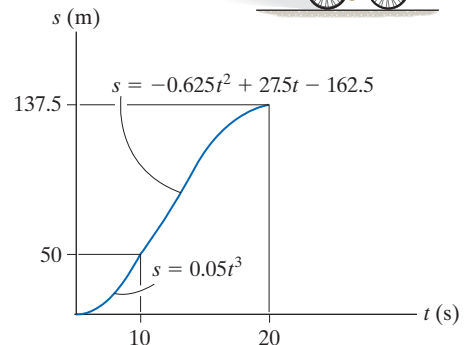
When $t = 0$ and 10 s,

$$a|_{t=0} = 0.3(0) = 0 \quad a|_{t=10 \text{ s}} = 0.3(10) = 3 \text{ m/s}^2$$

For the time interval $10 \text{ s} < t \leq 20$ s,

$$\left(\pm \right) \quad a = \frac{dv}{dt} = \frac{d}{dt} (-1.25t + 27.5) = -1.25 \text{ m/s}^2 \quad \text{Ans.}$$

The $a-t$ graph is shown in Fig. (b).



Ans:

For $0 \leq t < 10$ s,

$$v = (0.15t^2) \text{ m/s}$$

$$a = (0.3t) \text{ m/s}^2$$

For $10 \text{ s} < t \leq 20$ s,

$$v = (-1.25t + 27.5) \text{ m/s}$$

$$a = -1.25 \text{ m/s}^2$$

12–49.

The particle travels along a straight line with the velocity described by the graph. Construct the $a-s$ graph.

SOLUTION

$a-s$ Graph: For $0 \leq s < 3$ m,

$$\left(\pm \right) \quad a = v \frac{dv}{ds} = (2s + 4)(2) = (4s + 8) \text{ m/s}^2$$

At $s = 0$ and 3 m,

$$a|_{s=0} = 4(0) + 8 = 8 \text{ m/s}^2$$

$$a|_{s=3 \text{ m}} = 4(3) + 8 = 20 \text{ m/s}^2$$

For $3 \text{ m} < s \leq 6$ m,

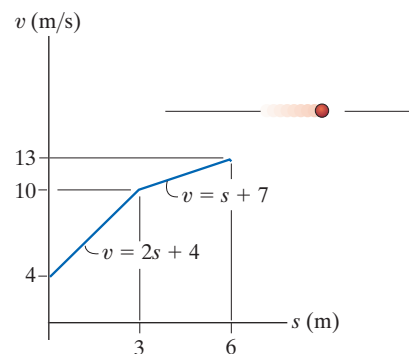
$$\left(\pm \right) \quad a = v \frac{dv}{ds} = (s + 7)(1) = (s + 7) \text{ m/s}^2$$

At $s = 3$ m and 6 m,

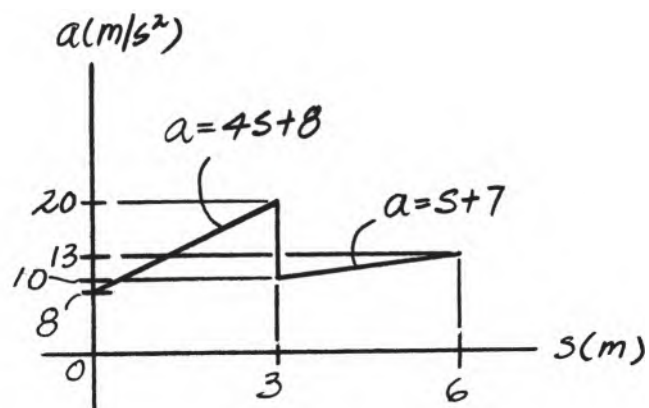
$$a|_{s=3 \text{ m}} = 3 + 7 = 10 \text{ m/s}^2$$

$$a|_{s=6 \text{ m}} = 6 + 7 = 13 \text{ m/s}^2$$

The $a-s$ graph is shown in Fig. (a).



Ans.



(a)

Ans:

For $0 \leq s < 3$ m,

$$a = (4s + 8) \text{ m/s}^2$$

For $3 \text{ m} < s \leq 6$ m,

$$a = (s + 7) \text{ m/s}^2$$

12-50.

The airplane travels along a straight runway with an acceleration described by the graph. If it starts from rest and requires a velocity of 90 m/s to take off, determine the minimum length of runway required and the time t' for takeoff. Construct the $v-t$ and $s-t$ graphs.

SOLUTION

$v-t$ Graph: For the time interval $0 \leq t < 10$ s, the initial condition is $v = 0$ when $t = 0$.

$$\begin{aligned} (\pm) \quad dv &= a dt \\ \int_0^v dv &= \int_0^t 0.8t dt \\ v &= (0.4t^2) \text{ m/s} \end{aligned}$$

Ans.

When $t = 10$ s,

$$v = 0.4(10^2) = 40 \text{ m/s}$$

For the time interval $10 \text{ s} < t \leq t'$, the initial condition is $v = 40$ m/s when $t = 10$ s.

$$\begin{aligned} (\pm) \quad dv &= a dt \\ \int_{40 \text{ m/s}}^v dv &= \int_{10 \text{ s}}^t 8 dt \\ v|_{40 \text{ m/s}}^v &= 8t|_{10 \text{ s}}^t \\ v &= (8t - 40) \text{ m/s} \end{aligned}$$

Ans.

Thus, when $v = 90$ m/s,

$$90 = 8t' - 40 \quad t' = 16.25 \text{ s} = 16.3 \text{ s} \quad \text{Ans.}$$

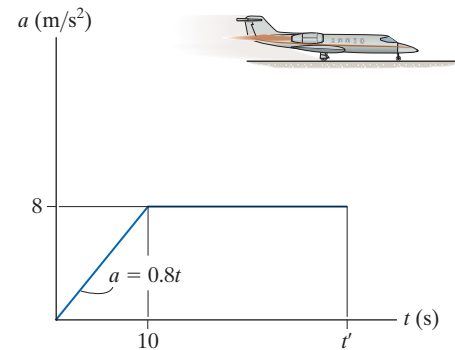
Also, the change in velocity is equal to the area under the $a-t$ graph. Thus,

$$\begin{aligned} \Delta v &= \int a dt \\ 90 - 0 &= \frac{1}{2}(8)(10) + 8(t' - 10) \\ t' &= 16.25 \text{ s} = 16.3 \text{ s} \end{aligned}$$

Ans.

Note: The result was rounded up to ensure $v \geq 90$ m/s.

The $v-t$ graph is shown in Fig. (a).



Free sample finished here!

Solution Manual for chapter 12 has 236 pages.

Last solved problem is #232.

