SOLUTIONS FOR INTERMEDIATE DYNAMICS
Second Edition

Patrick Hamill

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Chapter 1

CHAPTER ONE PROBLEMS

Problem 1.1. Two ships are sailing in a thick fog. Initially, ship A is 10 miles north of ship B. Ship A sails directly east at 30 miles per hour. Ship B sails due east at constant speed v_B then turns and sails due north at the same speed. After two hours, the ships collide. Determine v_B.

SOLUTION 1.1.

$$\begin{aligned} x_A(t) &= x_B(t) \\ y_A(t) &= y_B(t) \end{aligned}$$

where t = total time

Let t_1 = time boat 1 sails East and t_2 = time it sails North. Then

$$t = t_1 + t_2$$

So at time t we have

$$\begin{aligned} y_A &= 10 \\ y_B &= v_B t_2 \end{aligned}$$

and

$$\begin{aligned} x_A &= 30t \\ x_B &= v_B t_1 \end{aligned}$$

So

$$\begin{aligned} x_B + y_B &= x_A + y_A \\ v_B t_1 + v_B t_2 &= 30t + 10 \\ v_B t &= 30t + 10 \\ v_B &= 30 + 10/t \end{aligned}$$

But we know that $t = 2$ hours, so

$$v_B = 35 \text{ mph}$$

Problem 1.2. You carefully observe an object moving along the $x-$axis and determine that its position as a function of time is given by

$$x(t) = 2t - 3t^2 + t^3.$$

(a) What is the position at time $t = 2$ seconds?
(b) What is the velocity at time $t = 2$ seconds?
(c) What is the acceleration at time $t = 2$ seconds?
(d) How far did it travel between times $t = 0$ and $t = 2$ seconds? (Note: Distance, not displacement! It might be helpful to plot x vs t for $0 \leq t \leq 2$.)

Solution 1.2. (a) $x(t) = 2t - 3t^2 + t^3$ therefore, $x(t = 2) = 2(2) + 3(2^2) + 2^3 = 0$

(b) $v(t) = \frac{dx}{dt} = 2 - 6t + 3t^2 \therefore v(t = 2) = 2 - 12 + 12 = 2$ m/s.

(c) $a(t) = \frac{dv}{dt} = -6 + 6t \therefore a(t = 2) = -6 + 12 = 6$ m/s^2.

(d) A plot of $x = x(t)$ shows that as t goes from 0 to 2 sec, x goes from 0 to 0.385, then back to -0.385 and then returns to zero. Therefore the total distance is 0.385+2(0.385)+0.385 = 1.54 m. (Displacement is zero.)

Problem 1.3. The first train leaves the station and accelerates at a constant rate to its maximum speed of 100 km/hr, reaching this speed at a distance of 2 km from the station. Five minutes later, a second train leaves the station and accelerates to 100 km/hr in 4 km. What is the distance between the two trains when they both reach maximum speed?

Solution 1.3. At the instant train B reaches 100 km/hr, it will be 4 km from the station and train A will be at 2+$v(t - t_a)$ km from the station, where v is 100 km/hr and t_a is time for train A to accelerate to 100 km/hr.

From $d = \frac{1}{2}(v_f + v_i)t$ and the fact that $v_i = 0$, we deduce that $t_a = (2)(2)/100 = 1/25$ hour

Also, the total time, t, is 5 minutes plus the time it takes train B to reach 100 km/hr (namely 2/25 hour) so

$$t = \left(\frac{5}{60} + \frac{2}{25}\right)$$

Consequently

$$x_a = 2 + 100\left(\frac{5}{60} - \frac{1}{25}\right) = 14.33 \text{ km}$$

and

$$x_b = 4 \text{ km}$$

so distance between trains is

$$14.33 - 4 = 10.33 \text{ km}$$

Problem 1.4. A small helicopter is trying to land on a barge in the ocean. The propeller delivers an upward force of 36,000 N and the helicopter is observed to be descending at a constant safe speed of 3 m/s when it is 100 m above the barge. But suddenly there is a malfunction and the upward force is reduced to 30,000 N.

(a) What is the mass of the helicopter?

(b) What is the acceleration of the helicopter after the malfunction?

(c) Assume this acceleration is maintained constant during the final descent. What is the speed of the helicopter when it contacts the barge?

SOLUTION 1.4. (a) If acceleration = 0, Force up = Force down $\therefore 36000 = mg$ so $m = 3670$ kg.

(b) The force down is still 36000 N but force up = 30000 N. $\therefore F = ma$ yields $a = -6000/3670 = -1.63$ m/s^2.

(c) $2as = v_f^2 - v_i^2 \therefore (2)(-1.63)(-100) = v_f^2 - (3)^2 \therefore v_f = 18.3$ m/s

Problem 1.5. You travel a distance d in time t. (a) If you traveled at speed v_1 for half the time and at v_2 for the other half of the time, what is your average speed? (That is, the time average.) (b) If you travel at speed v_1 for half of the distance and at speed v_2 for the other half of the distance, what is your average speed?

SOLUTION 1.5. (a) $< v >= \frac{1}{t}\left[v_1\frac{t}{2} + v_2\frac{t}{2}\right] = \frac{v_1+v_2}{2}$

(b) $v=$ distance/time so time $=t = t_1 + t_2 = \frac{d/2}{v_1} + \frac{d/2}{v_2} = \frac{d}{2}\left(\frac{v_1+v_2}{v_1 v_2}\right)$

$\therefore < v >= \frac{d}{d/2}\frac{v_1 v_2}{v_1+v_2} = \frac{2v_1 v_2}{v_1+v_2}$

Problem 1.6. There is a long straight road out in the desert and it goes through a small town that has just one police car. The police car accelerates at 2 m/sec^2 until it reaches a maximum speed of 200 km/hour. A car full of escaped criminals speeds through the town at its top speed which is 150 km/hour. The police car, starting from rest, gives chase. How far from the town do the police catch up to the criminals?

SOLUTION 1.6. At the moment the police catch up to the criminals, the distances and total times are equal for the two automobiles.

Let t_c= total time for criminals, taveling at 150 km/hr = 41.67 m/s = v_c, and let $t_1 + t_2$ be the total time for the police, where t_1 is the time the police car is accelerating, and t_2 is the time the police car is traveling at constant speed of 200 km/hr =55.56 m/s = v_2.

Let d_c be the distance traveled by the criminals noting that $d_c = v_c t_c$ and let the distance traveled by the police car be $d_1 + d_2$ where d_1 is given by $2a_1 d_1 = v_f^2$ and $d_2 = v_2 t_2$.

Note that we can evaluate $d_1 = v_f^2/2a = (55.6)^2/(2 * 2) = 772m$, where v_f is the final speed reached by the police car = 55.6 m/s. Furthermore, since $v_f = at_1$ we can determine t_1 = the time the police car was accelerating. This is $t_1 = 55.6/2 = 27.8$ s

So we known d_1 and t_1. From $d_c = d_1 + d_2$ and $t_c = t_1 + t_2$ we have

$$41.7t_c = 772+55.6(t_2) = 772+55.6(t_c-t_1) = 772+55.6(t_c)-55.6(27.8)$$

That is

$$t_c(41.7 - 55.6) = 772 - 55.6(27.8)$$

and so $t_c = 55.5s$. Therefore the distance from town $=d_c = v_c t_c = 41.67(55.6) = 2315$ m or 2.3 km.

Problem 1.7. A police car is at rest at the side of the road when a wild teenager comes speeding by at 75 miles per hour. The police car starts immediately and accelerates at 8 miles per hour per second. At that same moment the teenager steps on the gas, but his car only accelerates at 2 miles per hour per second. How far from the starting point does the police car overtake the speeder? How fast are they going at that time? Why is the speed you calculated for the police car unrealistic?

SOLUTION 1.7. Changing miles per hour to feet per second (60 mph = 88 ft/s) we have initial speed of teenager = 110 ft/s and the accelerations are a_1 = 2 mph/s= 2.93 ft/s^2 and a_2 = 8 mph/s= 11.73 ft/s^2. When the police car catches up, the positions of the two cars are the same, and the time is the same, so

$$\begin{aligned} d_1 &= d_2 \\ \frac{1}{2}a_1 t^2 + v_0 t &= \frac{1}{2}a_2 t^2 \\ a_1 t + 2v_0 &= a_2 t \end{aligned}$$

or

$$\begin{aligned} t(a_1 - a_2) &= 2v_0 \\ t &= \frac{2(110)}{11.73 - 2.93} = 25 \text{ s.} \end{aligned}$$

The distance traveled is

$$d = \frac{1}{2}a_2t^2 = 0.5(11.73)(25)^2 = 3665 \text{ ft} = 0.69 \text{ mi}$$

and the speeds of the two cars are:

$$v_1 = v_0 + a_1t = 110 + 2.93(25) = 183 \text{ ft/s} = 125 \text{ mph},$$

$$v_2 = a_2t = 11.73(25) = 293.3 \text{ ft/s} = 200 \text{ mph}.$$

The speed of the police car is unrealistically large because we assumed it could accelerate at a constant rate. This is not possible, as acceleration drops off as the speed increases.

Problem 1.8. A brick is on a wooden plank that is resting on a table. One end of the plank is slowly raised so that it forms an angle θ with the horizontal table top. When $\theta = 60°$ the brick starts to slide down the plane. (a) Draw the free body diagram. (b) Determine the coefficient of static friction between brick and plank. (c) If the coefficient of sliding friction is one half of the coefficient of static friction, determine the acceleration of the block.

SOLUTION 1.8. (b) $\mu N = mg\sin\theta$ and $N = mg\cos\theta \therefore \mu = \frac{mg\sin\theta}{mg\cos\theta} = \tan\theta = \tan 60° = 1.73$.

(c) Force down plane $= mg\sin\theta - \mu mg\cos\theta = ma$ Therefore, $a = g(\sin\theta) - g(1.73/2) = 9.8(0.866 - 0.43) = 4.24m/s^2$

Problem 1.9. A block of mass M is on an inclined plane with coefficient of sliding friction μ. At a given instant of time the block is located at some point on the plane and is moving up the plane with a speed v_0. (a) Obtain the time for the block to reach its highest point. (b) Obtain the time for the block to slide back down to its starting point. (c) Obtain an expression for the velocity of the block at the time it returns to the starting point.

SOLUTION 1.9. The block moves up the plane until $v = 0$. The acceleration of the block can be obtained by drawing a free body diagram. The forces on the block parallel to the plane are friction, given by

$\mu N = \mu Mg\cos\theta$ and a component of the gravitational force, $Mg\sin\theta$. From $F = Ma$ we obtain

$$\begin{aligned} Ma &= -Mg\sin\theta - \mu N \\ a &= -g\sin\theta - \mu g\cos\theta = -g(\sin\theta + \mu\cos\theta). \end{aligned}$$

From $2as = v_f^2 - v_0^2$ we obtain

$$-2g(\sin\theta + \mu\cos\theta)s = 0 - v_0^2$$

and so the distance it goes up the plane (s) is given by

$$s = \frac{v_0^2/2g}{\sin\theta + \mu\cos\theta}.$$

The time to stop is given by $v_f = v_0 + at$, or

$$0 = v_0 - g(\sin\theta + \mu\sin\theta)t$$

Call this time t_{top}. That is

$$t_{top} = \frac{v_0/g}{\sin\theta + \mu\cos\theta}.$$

(b) Now the block slides back down the plane. During this part of the motion the gravitational force still has a component down the plane, but the frictional force is up the plane. You can easily show that the acceleration is

$$a = g\sin\theta - \mu g\cos\theta.$$

The time to go a distance s is obtained from

$$s = (1/2)at^2,$$

since it started from rest. But we have an expression for s and we have an expression for a, so subbing we obtain

$$\frac{v_0^2/2g}{\sin\theta + \mu\cos\theta} = \frac{1}{2}\left(g\sin\theta - \mu g\cos\theta\right)t^2.$$

Call this time to slide back down, t_{bot}, that is

$$\begin{aligned} t_{bot}^2 &= \frac{\frac{v_0^2/g}{\sin\theta+\mu\cos\theta}}{g(\sin\theta - \mu\cos\theta)} = \frac{v_0^2/g^2}{(\sin\theta + \mu\cos\theta)(\sin\theta - \mu\cos\theta)} \\ &= \frac{v_0^2/g^2}{\sin^2\theta - \mu\cos^2\theta}. \end{aligned}$$

(c) The velocity at the bottom is given by

$$2as = v_f^2 - v_0^2$$

and now $v_0 = 0$ and $a = g\sin\theta - \mu g\cos\theta$ and $s = \frac{v_0^2/2g}{\sin\theta+\mu\sin\theta}$, so

$$\frac{2g(\sin\theta - \mu\cos\theta)(v_0^2/2g)}{\sin\theta + \mu\cos\theta} = v_f^2$$

or

$$v_f = v_0\sqrt{\frac{\sin\theta - \mu\cos\theta}{\sin\theta + \mu\cos\theta}}.$$

Problem 1.10. A railgun is a device that uses electromagnetic forces to accelerate a body along a set of conducting rails. Assume a railgun accelerates some object directly upward at 60 m/s^2 for 1.5 seconds. The object then coasts upwards to some maximum altitude before falling back down. Determine the maximum altitude reached. Ignore air resistance.

SOLUTION 1.10. a=60 m/s^2 and $t = 1.5$ seconds, so velocity as the body leaves the railgun is $v = (1/2)at^2$=67.5 m/s. It then rises to a distance s while accelerating downwards at 9.8 m/s^2. Using $2as = v_f^2 - v_i^2$ we have $s = (67.5)^2/(2 \times 9.8) = 232$ m.

Problem 1.11. Atwood's machine consists of two weights (M_1 and M_2) suspended at the ends of a string that passes over a pulley. Assume massless, inextensible strings and a frictionless pulley. Let $M_1 = 6$ kg and $M_2 = 5.5$ kg. The masses are released from rest. Determine the distance descended by the 6 kg mass when its velocity reaches 0.5 m/s.

SOLUTION 1.11. Apply $F = ma$ to the two bodies:

$$\begin{aligned} T - M_1 g &= M_1 a \\ T - M_2 g &= -M_2 a \end{aligned}$$

Subtract to get $a = g\frac{M_2 - M_1}{M_2 + M_1} = g\frac{5-6}{5+6} = -1/11g$

Then, using $2as = v_f^2 - v_i^2 = (2)(-1/11)(s) = (0.5)^2 - 0$ we obtain $s = 1.38$ meters.

Problem 1.12. (a) Determine the rotational kinetic energy of a wheel of your bicycle when your linear speed is 20 km/hour. You may assume the wheel is a hoop of mass 1.5 kg and radius 30 cm. (b) Compare your result with the translational kinetic energy of the wheel. (c) Is the equality of parts (a) and (b) just a numerical coincidence or is it always true? (d) Would the energies be equal if the wheel were a disk rather than a hoop?

SOLUTION 1.12. (a) $v = \omega r$. v= 20 km/hr = 5.56 m/s. $\therefore, \omega = 18.52$ rad/s and the rotational kinetic energy is

$$T_R = \frac{1}{2} I\omega^2 = \frac{1}{2}(mr^2)\omega^2 = 23.15J.$$

(b) The translational kinetic energy is

$$T_T = \frac{1}{2} mv^2 = \frac{1}{2}(15)(5.56)^2 = 23.15J.$$

(c) The two energies will always be the same because

$$\frac{T_R}{T_T} = \frac{(1/2)I\omega^2}{(1/2)mv^2} = \frac{(1/2)(mr^2)(v/r)^2}{(1/2)mv^2} = \frac{(1/2)mv^2}{(1/2)mv^2} = 1.$$

(d) The equality would not hold if the wheel were a disk because the moment of inertia of a disk is $\frac{1}{2}mr^2$ so $T_R/T_T = \frac{1}{2}$

Problem 1.13. The frictional force between water and seabed in shallow seas cause an increase in the day by about 1 ms/century. Determine the torque that causes this change. Assume the Earth is a sphere.

SOLUTION 1.13. If the period (T) increases, the angular speed (ω) decreases. Specifically,

$$\omega = \frac{2\pi}{T}$$
$$\therefore \quad d\omega = -\frac{2\pi}{T^2} dT$$

Let us write this as

$$\Delta\omega = -\frac{2\pi}{T^2}\Delta T.$$

Now $\Delta T = 10^{-3}$ seconds and the time for the Earth to spin through an angle of 2π is one day or 8.64×10^4 seconds. Therefore,

$$\Delta\omega = -\frac{2\pi}{(8.64 \times 10^4)^2} 10^{-3} = -8.42 \times 10^{-13} \text{ rad/sec.}$$

The angular acceleration is

$$\alpha = \frac{-8.42 \times 10^{-13}}{\Delta t},$$

where Δt is the time during which the slowing down process takes place, namely one century, or $100 \times 365.24 \times 24 \times 3600$ seconds. Therefore

$$\alpha = \frac{\Delta\omega}{100 \times 365.24 \times 24 \times 3600} = -2.67 \times 10^{-23} \text{ rad/sec/sec.}$$

The torque is given by

$$N = I\alpha$$

where $I = \frac{2}{5}MR^2 = \frac{2}{5}(5.97\times 10^{24})(6.37\times 10^6)^2 = 9.69\times 10^{27}$. Therefore

$$N = \left(9.69 \times 10^{27}\right)\left(2.67 \times 10^{-23}\right) = 2.58 \times 10^{16} \text{ N m}.$$

Problem 1.14. A meter stick has a pivot at one end. It is found to be in static equilibrium when acted upon by three forces that act at different points along the meter stick and act in different directions. We conclude that the net torque about the pivot is zero. Prove that the net torque about any other (arbitrary) point is also zero.

Solution 1.14. Let the three forces be $\mathbf{F_1}, \mathbf{F_2}, \mathbf{F_3}$ and assume they are acting at points $\mathbf{r_1}, \mathbf{r_2}, \mathbf{r_3}$ measured from the pivot. According to the statement of the problem, the torque about the pivot is zero so

$$0 = \mathbf{N_{tot}} = \mathbf{r_1} \times \mathbf{F_1} + \mathbf{r_2} \times \mathbf{F_2} = \mathbf{r_3} \times \mathbf{F_3}$$

Let A represent an arbitrary point a distance $\mathbf{d}$ from the pivot. Let the vectors from A to the points of application of the forces be $\mathbf{l_1}, \mathbf{l_2}, \mathbf{l_3}$. Then the torque about A is

$$\mathbf{N_A} = \mathbf{l_1} \times \mathbf{F_1} + \mathbf{l_2} \times \mathbf{F_2} + \mathbf{l_3} \times \mathbf{F_3}.$$

But $\mathbf{l_1} = \mathbf{r_1} - \mathbf{d}$ and similarly for $\mathbf{l_2}, \mathbf{l_3}$, so

$$\begin{aligned} \mathbf{N_A} &= (\mathbf{r_1} - \mathbf{d}) \times \mathbf{F_1} + (\mathbf{r_2} - \mathbf{d}) \times \mathbf{F_2} + (\mathbf{r_3} - \mathbf{d}) \times \mathbf{F_2} \\ &= \mathbf{r_1} \times \mathbf{F_1} + \mathbf{r_2} \times \mathbf{F_2} = \mathbf{r_3} \times \mathbf{F_3} - \mathbf{d} \times \mathbf{F_1} - \mathbf{d} \times \mathbf{F_2} - \mathbf{d} \times \mathbf{F_3} \\ &= 0 + \mathbf{d} \times (\mathbf{F_1} + \mathbf{F_2} + \mathbf{F_3}) \\ &= 0, \end{aligned}$$

where we used the fact that if the body is in equilibrium the sum of all the forces acting on it must be zero.

Problem 1.15. To build the pyramids it was necessary to pull heavy stones up inclined planes. Suppose a 2000 kg stone was dragged up a 20° incline at a speed of 0.25 m/s by a gang of 20 laborers. The coefficient of kinetic friction between the stone and the incline was 0.4. How much power was exerted by each laborer?

Solution 1.15. From a free body diagram, the force exerted on the stone has to be

$$\begin{aligned} F &= mg\sin\theta + \mu mg\cos\theta \\ &= (2000)(9.8)(\sin 20^\circ + 0.4\cos 20^\circ) \\ &= 14000N. \end{aligned}$$

The power is given by

$$P = Fv = 3520 \text{ watt.}$$

Divide by 20 laborers to get 176 watts per person.

Problem 1.16. Two wooden blocks, M_1 and M_2 are sliding in the same direction at the same speed on a frictionless horizontal surface. Let $M_1 = M_2 = 3$ kg and assume their speed is 2 m/s. A third block of mass $m = 1$ kg is also sliding in the same direction at a speed of 10 m/s, and it collides with the trailing 3 kg block. The third block is covered with a sticky, gooey substance, so it sticks to the trailing block. This combination block catches up with and collides elastically with the leading 3 kg block. Determine the final speed of the leading block. (Note that this is a one-dimensional problem. You may find it easier to solve the problem if you insert numerical values sooner rather than later.)

Solution 1.16. The first collision is inelastic. By conservation of momentum

$$mv + M_1V_1 = (m + M_1)V_c = M_cV_c.$$

So the speed of the combination block (V_c) is given by

$$V_c = \frac{mv + M_1V_1}{m + M_1} = \frac{1(10) + 3(2)}{1 + 3} = 4 \text{ m/s.}$$

The combined block has mass 4 kg and speed 4 m/s. It collides elastically with the leading block. Conservation of momentum gives

$$M_cV_c + M_2V_2 = M_cV_c' + M_2V_2',$$

where the primes indicate final speeds. Inserting numerical values we have

$$\begin{aligned} 4(4) + 3(2) &= 4V_c' + 3V_2' \\ &\therefore \;\; V_2' = \frac{1}{3}(22 - 4V_c'). \end{aligned}$$

Since this collision is elastic we can use conservation of energy to write

$$\begin{aligned} \frac{1}{2}M_cV_c^2 + \frac{1}{2}M_2V_2^2 &= \frac{1}{2}M_c(V_c')^2 + \frac{1}{2}M_2(V_2')^2 \\ 4(4)^2 + 3(2)^2 &= 4(V_c')^2 + 3(V_2')^2 \\ &\therefore \;\; V_c'^2 = \frac{1}{4}\left(76 - 3(V_2')^2\right). \end{aligned}$$

Substituting into the expression for V_2' we get

$$\begin{aligned} V_2' &= \frac{1}{3}\left(22 - 4\sqrt{\frac{1}{4}\left(76 - 3(V_2')^2\right)}\right) \\ V_2' - \frac{22}{3} &= -\frac{2}{3}\sqrt{76 - 3(V_2')^2}. \end{aligned}$$

Squaring both sides

$$\begin{aligned} (V_2')^2 - \frac{44}{3}V_2' + \left(\frac{22}{3}\right)^2 &= \frac{4}{9}\left[76 - 3(V_2')^2\right] \\ \frac{7}{3}V_2'^2 - \frac{44}{3}V_2' + \frac{22^2 - 4(76)}{9} &= 0 \\ V_2'^2 - 6.3V_2' + 8.6 &= 0. \end{aligned}$$

This quadratic has the solutions

$$V_2' = 4.3 \text{ and } 2.0.$$

From the statement of the problem we see that the second solution is just the initial velocity of the block, so the answer to our problem is

$$V_2' = 4.3 \text{ m/s}.$$

Problem 1.17. You are driving at 60 mph (=100 km/hr). Your car's wheels have a radius of 35 cm. (a) Determine the angular velocity of the wheels. (b) What is the angular displacement of a wheel when you travel 1 km? (c) If you slow down and stop in 1 km, what is the angular acceleration of the wheel?

Solution 1.17. 100 km/hr = 27.78 m/s

(a) $\omega = v/r = 27.78/0.35 = 79.37$ rad/s.

(b) Each turn of the wheel advances it by 2πr=2.20 m. Therefore, in advancing 1 km the wheel turns n times where n = 1000/2.20=455 turns. Consequently,

$$\theta = 2\pi n = 2\pi(455) = 2860 \text{ rad}$$

(c) $2\alpha\theta = \omega_f^2 - \omega_i^2$. Therefore,

$$2\alpha(2860) = 0 - (79.37)^2$$

and $\alpha = -1.10$ rad/s^2.

Problem 1.18. Aeronautical engineers have developed "tip jet" helicopters in which small jet engines are attached to the tips of the rotor. One such helicopter is powered by two ramjets. For a ramjet to develop thrust, it needs to be moving through the air quite rapidly, and is not efficient until it is moving at about 1000 km/hr. Assume the rotor diameter is 10 meters. Determine the angular speed of the rotor when the ramjet is moving through the air at 1000 km/hr.

SOLUTION 1.18. $\omega = v/r = (1000)(1000/3600)(1/5) = 55.6$ rad/s

Problem 1.19. Figure 1.8 shows an object of mass M that is hanging from a pivot at point P. The three segments have equal length. (a) Show that the object is in equilibrium. (b) Determine whether or not this is a stable equilibrium orientation.

SOLUTION 1.19. (a) Consider the three segments of the object; call them "left", "middle", and "right." Let each segment have length l. The mass of each segment is $M/3$. The torque about the pivot is the sum of the torques on each segment. Using the formula, torque = force × lever arm we appreciate that the torque about the pivot by "left" segment is counterclockwise (CCW) and given by $g(M/3)(l/2) = Mgl/6$. The torque about the pivot by the "middle" segment is zero because it has zero lever arm. The torque about the pivot by the "right" segment is clockwise (CW) and equal to $Mgl/6$). Therefore, the net torque about the pivot is zero.

(b) This is a stable equilibrium because if it is disturbed by a small CCW rotation, the middle segment develops a non-zero lever arm and a CW torque. Also, the lever arm of the "left" segment gets a little bit smaller and the lever arm of the "right" segment gets a little bit larger. Consequently, there is a net restoring torque about the pivot.

Problem 1.20. A solid ball of mass M and radius R rolls down an inclined plane. (a) What is its translational speed when it has descended a vertical distance h? (b) Determine its translational kinetic energy and its rotational kinetic energy.

SOLUTION 1.20. Final kinetic energy equals initial potential energy $\therefore$, $T_f = Mgh$ where the final kinetic energy T_f is the sum of the translational kinetic energy T_t and the rotational kinetic energy T_r.

$$\therefore Mgh = T_t + T_r = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}Mv^2 + \frac{1}{2}(\frac{2}{5}MR^2)\frac{v^2}{r^2}$$

so

$$mgh = \frac{1}{2}Mv^2 + \frac{1}{5}Mv^2 = \frac{7}{10}Mv^2$$

That is, $v = \sqrt{\frac{gh}{0.7}}$

(b) The translational kinetic energy is $T_t = \frac{1}{2}Mv^2 = \frac{1}{2}Mg\frac{gh}{0.7} = \frac{10}{14}Mgh$

The rotational kinetic energy is $T_r = Mgh - \frac{10}{14}Mgh = \frac{4}{14}Mgh$.

Problem 1.21. A disk of mass 72 kg and radius 50 cm is rotating at 2000 rpm. (a) Determine its angular momentum. (b) If acted upon by a retarding force of 20 N acting tangent to the rim of the disk, determine the time required to stop the disk.

SOLUTION 1.21. (a)$L = I\omega$ and $\omega = 2\pi 2000/60 = 209$ rad/s. The moment of inertia is $I = \frac{1}{2}MR^2 = \frac{1}{2}(72)(0.5)^2 = 9$

Therefore, $L = I\omega = (9)(209) = 1885 kgm^2/sec$

(b) $N = \frac{dL}{dt}$. The torque is $N = 20 \times 0.5 = 10$ Nm. Since torque is constant, $\Delta L = Nt = 10t \therefore, t = 1885/10 = 186$ seconds.

Computational Project 1.1. The position of a particle as a function of time is given by $x = 5t^3 - 2t$ (meters). Plot the position as a function of time for the interval $t = -5$ sec to $t = +10$ sec. Using the relationship $\overline{v} = \Delta x/\Delta t$, obtain the average velocity at one second intervals. Plot the average velocity as a function of time; on the same graph, plot the analytical expression for the velocity.

PROGRAM 1.1. Matlab Program

```
%cp1.1
for i=1:15
    t(i)=-5+(i-1);
    x(i)=5*t(i)^3-2*t(i);
end
for i=1:14
    midtime(i)=(t(i+1)+t(i))/2;
    vbar(i)=x(i+1)-x(i);
    v(i)=15*midtime(i)^2-2;
    end
subplot(2,1,1)
plot(t,x)
xlabel('time (sec)')
ylabel('position (meters)')
title('Position vs time')
subplot(2,1,2)
plot(midtime,vbar,'r+',midtime,v,'b.')
xlabel('time (sec)')
ylabel('av vel (red) and velocity (blue)')
title('Velocity vs time')
```

Computational Project 1.2. The velocity of a certain particle is given by

$$v = 120(1 - e^{-t/10}) + 0.5\cos(t/2)$$

in meters per second. Plot the velocity as a function of time. Determine and plot the distance as a function of time by numerically adding up the area under the velocity vs. time curve.

PROGRAM 1.2. Solution:

```
%cp1_2 Velocity of particle and numerical integration
clear
delta_t=0.1;
t(1)=delta_t;
v(1)=120*(1-exp(-t(1)/10.0))+0.5*cos(t(1)/2);
d(1)=v(1)*delta_t; %Area under curve
N=input('input number of iterations, like 1000 ')
for i=2:N;
t(i)=t(i-1)+delta_t;
v(i)=120*(1-exp(-t(i)/10.0))+0.5*cos(t(i)/2);
d(i)=v(i)*delta_t; %Area under curve
end
totdis(1)=d(1);
for i=2:N
totdis(i)=totdis(i-1)+d(i);
end
subplot(1,2,1)
plot(t,v);
title('velocity vs time')
xlabel('time (arbitrary units)');
ylabel('velocity (arbitrary units)');
subplot(1,2,2)
plot(t,totdis);
title('distance vs time');
xlabel('time (arbitrary units)');
ylabel('position (arbitrary units)');
```

Computational Project 1.3. This is a more realistic version of problem 1.7. In that problem a teenager driving at 33.5 m/s (∼75 mph) speeds past a parked policeman. The policeman and the teenager then accelerate at given constant rates and you are required to determine how far from the starting point the policeman catches up to the teenager. To make the problem somewhat more realistic, assume the teenager accelerates at $a_T = k_T e^{-b_T t}$ and the policeman accelerates at $a_P = k_P e^{-b_P t}$, where $k_T = 5$ mph/s and $k_P = 10$ mph/s. By plotting the positions as functions of time for various values of b_T and b_P obtain reasonable values for these constants. (Make sure your answers are reasonable. Eventually the policeman will catch the teenager, but your answer is not reasonable if the distance required is hundreds of miles!)

PROGRAM 1.3. Solution:

```
%cp1_3
clear
bt=input('input value of b_t, like 0.1 ');
bp=input('input value of b_p, like 0.05 ');
dt=0.05; %time step = 1/20 second
t(1)=0; %initial time = zero at(1)=0; %initially neither car is accelerating
ap(1)=0;
vt(1)=33.5; %speed of teenager at initial time
vp(1)=0.0; %speed of policeman at initial time
xt(1)=0.0; %initial positions are set to zero
xp(1)=0.0;
kt=2.24; %Note 5 mph/sec = 2.24 m/s^2
kp=4.48; %Note 10 mph/s= 4.48 m/s^2
for i=1:2000
t(i)=t(i-1)+dt;
at(i)=kt*exp(-bt*t(i));
ap(i)=kp*exp(-bp*t(i));
end
for i=2:2000
vt(i)=vt(i-1)+at(i-1)*dt;
vp(i)=vp(i-1)+ap(i-1)*dt;
xt(i)=xt(i-1)+vt(i-1)*dt;
xp(i)=xp(i-1)+vp(i-1)*dt;
end
%final speed of policeman must be realistic...in miles per hour it is
vpp=vp(1000)*2.24
subplot(2,1,1)
plot(t,xt,'r',t,xp,'b')
xlabel('time (sec)')
ylabel('distance (meters)')
title('Distances for Teenager (red) and Policeman (blue)')
subplot(2,1,2)
plot(t,vt,'r',t,vp,'b')
xlabel('time (sec)')
ylabel('speeds (meters/sec)')
title('Speeds for Teenager and Policeman')
```

Computational Project 1.4. A large number of electrons are randomly distributed in a small region of space. (The dimension is not important, but you can assume a cube with side 10 angstroms.) Numerically determine the position of the center of mass, for 100, 1000, and 10000 electrons. (You should use a random number generator to obtain the coordinates of each of the electrons.)

PROGRAM 1.4. Solution:

```
%cp1_7
n=100;
for i=1:n
     x(i)=rand(1);
     y(i)=rand(1);
     z(i)=rand(1);
end
sumx=0;
sumy=0;
sumz=0;
for i=1:n
sumx=sumx+x(i);
sumy=sumy+y(i);
sumz=sumz+z(i);
end
cmx=sumx/n
cmy=sumy/n
cmz=sumz/n
n=1000
for i=1:n
     x(i)=rand(1);
     y(i)=rand(1);
     z(i)=rand(1);
end
sumx=0;
sumy=0;
sumz=0;
for i=1:n
sumx=sumx+x(i);
sumy=sumy+y(i);
sumz=sumz+z(i);
end
cmx=sumx/n
cmy=sumy/n
```

```
cmz=sumz/n
n=10000
for i=1:n
    x(i)=rand(1);
    y(i)=rand(1);
    z(i)=rand(1);
end
sumx=0;
sumy=0;
sumz=0;
for i=1:n
sumx=sumx+x(i);
sumy=sumy+y(i);
sumz=sumz+z(i);
end
cmx=sumx/n
cmy=sumy/n
cmz=sumz/n
```